

NUCLEAR MODELS

By

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Preface

The study of nuclear physics mainly aims at an understanding of the structures of atomic nuclei. Its elucidation could be made possible by a comprehensive grasp of nuclear spectroscopy which could be interpreted by models formulated by employing the characteristic data of nuclei under different conditions. The earliest attempt has been made under the caption of the extreme single particle model which with its modification describes the nuclear state by stating the number of nucleons in each one of the orbits, the number of degenerate substates for each level corresponding to different orientations of angular momentum.

The shell model though seemed satisfactory, has been found to have a measure of approximation. Hence arose the necessity of formulating the individual particle model involving a method of obtaining nuclear wave functions which though in principle can be quite accurate depends on the approximate validity of the shell model. The measure of approximation necessitates to widen the scope of the shell model by formulating the individual particle model which involves a method of obtaining nuclear wave functions. This model takes for wave functions completely antisymmetrized products of single particle states for each nucleon described by $L-S$ and $j-j$ coupling scheme; in former the single particle wave functions being taken as describing particles of fixed orbital and spin angular momenta and fixed charge. Employing the procedure with elaboration and choosing suitable set of wave functions, the potential energy is used to describe their interaction. With these considerations it was found reasonable to modify the two nucleon potential for application to the independent particle model. The results of independent particle model calculations are energy levels predicted for nuclei which are in agreement with the experimental results.

The shell and the individual particle models on the one hand and the strong interaction or the liquid drop model on the other have distinct approaches to explain nuclear structure. They cover the region of certain types of nuclear phenomena.

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with distinct regions of applicability. Then arose the necessity for the development of a model capable of retaining the distinct features giving rise to the evolution of a model which is capable of describing the nucleus as a shell-structure as in single particle and independent particle models with the addition of static isotropic nuclear field. In this field the motion of nucleons was substituted by a nuclear potential which allows for variation in nuclear field resulting from the oscillations in shape, and changes in the special orientations of the nucleons. The particle motions and the nuclear oscillations, mutually effect each other. It is then required to compute the interaction of these two intrinsically different types of motion and find out their effects on the nuclear characteristics such as spin, parity, static and dynamic electromagnetic moments. This procedure gives rise to the nuclear model referred to as the unified model which when employed exclusively for the description of spheroidal nuclei, exhibiting characteristic rotation spectra, is termed as the rotational model and that in which when the collective motions in the nuclei as a whole are taken into account is termed as the collective model.

The importance of the problem of the many particle wave functions of odd-A nuclei, odd-odd nuclei, even-even nuclei as well as for cases of quadrupole vibrations is dealt with along with microscopic theory of collective motions and other important cases. Thus the collective model paves the way for the interpretation of experimental nuclear spectra.

There are cases especially in odd-A spherical nuclei which exhibit such characteristics that cannot be covered by the interpretation of the models that have been already dealt with. Another set of models which so to speak can comprehensively interpret the spectra has been found necessary so that low energy excited states in such nuclei were proposed to be viewed as arising out of core-excitations coupled to the particle in the various single particle orbits. Thus the core excitation multiplets are given rise to the transition probabilities among which appear enhancements over single particle estimates.

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The author in writing this work has employed a great deal of information which has been culled from the vast mass of the existing literature, scientific periodicals and journals. His work and studies as well as many fruitful discussions he has had with his colleagues, research scholars and student friends have been a guiding force in tackling with a measure of satisfaction in dealing with the topics.

He also mentions his deep appreciation of the help that has been rendered to him in the preparation of the manuscript press copy to his friends Dr. V. Lakshminarayana, Dr. S. Rama Murty, Dr. K. Venkata Reddy, Dr. D. Lakshmana Sastry and especially to Dr. K. Parthasaradhi.

In concluding this preface the author takes the opportunity of expressing his most grateful and heartfelt thanks to Prof. K. R. Srinivasa Iyengar, M.A., D.Litt., Vice-Chancellor of the Andhra University for his unstinted support and encouragement in the endeavours of the author by initiating the publication of this work by the Andhra University.

Finally it will not be out of place if the author mentions his deep debt of gratitude to Sri P.V.G. Raju, Pro-Chancellor of the Andhra University for assisting in the publication of this work by the Andhra University.

Waltair

SWAMI JNANANANDA

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Single Particle Model

1.1 Preliminary Remarks

Of all the nuclear models that are formulated to explain characteristics of all aspects of the phenomena of nuclear structure, single particle model is perhaps the earliest one. In this model, the individual nucleons are treated as moving in stationary orbits and pairing off in such a way that the values of several parameters are determined entirely by a single unpaired nucleon in the case of an odd mass nucleons. The model does not involve either correlated or collective motion of several nucleons. Nor does it refer explicitly to two body forces between nucleons. The afore mentioned, especially the latter tend to render it of limited applicability in so far as its predictions of quadrupole moments have turned out to be wrong. A description of quadrupole moments within framework of a model other assumptions are made such as the non-spherical distorted liquid drop like nucleus without the motion of nucleons. Even so it could hardly predict the discontinuities associated with the magic numbers. Hence a model must be considerably more complicated than either of the two mentioned extreme cases. Hence the description of models which can cover the actual aspects of the phenomena of nuclear structure may be treated as a generalisation and extension formulated on the single particle model. The single particle orbits may represent an average of the actual nucleon motions. Certain nuclear quantities are sensitive to the average motion. There are, however, others which are strongly affected by the details of nuclear structure, especially, by particle correlations. To begin with, the description of single particle model which

motions. Hence although the center loses its physical significance, it may be assumed as one which coincides with mass center. If the center of the potential is fixed at the origin there can be oscillations of the mass about the origin. The energy associated with the motion of the mass center is rather imaginary since it is only the mass center that is actually fixed.

In the case of central potential, the orbital angular momentum of each nucleon is a constant of motion. The orbital quantum number l is related to the radial quantum number n , associated with the number of nodes of the radial wave function. Let the state of lowest energy with $l=1, l=3, \dots$ be called $1p, 1f, \dots$ states respectively. The spacing of the energy levels depends on the form of the potential. But the order of levels is almost fixed for the general type of potential $V(r)$. The harmonic oscillator and square well types of potentials which represent the two extreme cases are given by

$$V(r) = \begin{cases} -V_0 [1 - (r/R)^2] & \text{for } r < R \\ 0 & \text{for } r > R \end{cases} \quad \dots (1.1)$$

and

$$V(r) = \begin{cases} -V_0 = \text{Const.} & \text{for } r < R \\ 0 & \text{for } r > R \end{cases} \quad \dots (1.2)$$

In the case of infinite square well, if $r > R$, the potential can be set at $V = +\infty$ instead of zero it is reasonable to disregard the exponentially decreasing tail of the wave functions outside the nucleus and replace it by $\psi = 0$. Hence equation 1.2 is reduced to

$$V(r) = V_0 = \text{const. for } r < R$$

whereas the harmonic oscillator potential

$$V(r) = -V_0 \{1 - (r/R)^2\}$$

can be retained for $r > R$ inasmuch as the exponentially decreasing tail of the wave function outside the nucleus is replaced by one that is more rapidly dropping down to zero.

The computed numerical values for harmonic oscillator V_0 for $r > R$ and for a square well potential of Fermi distribution type⁽⁶⁾ may yield same level orders as well as those of intermediate shape as represented in Fig. 1.1. It can be noted that the levels of the harmonic oscillator are evenly spaced and

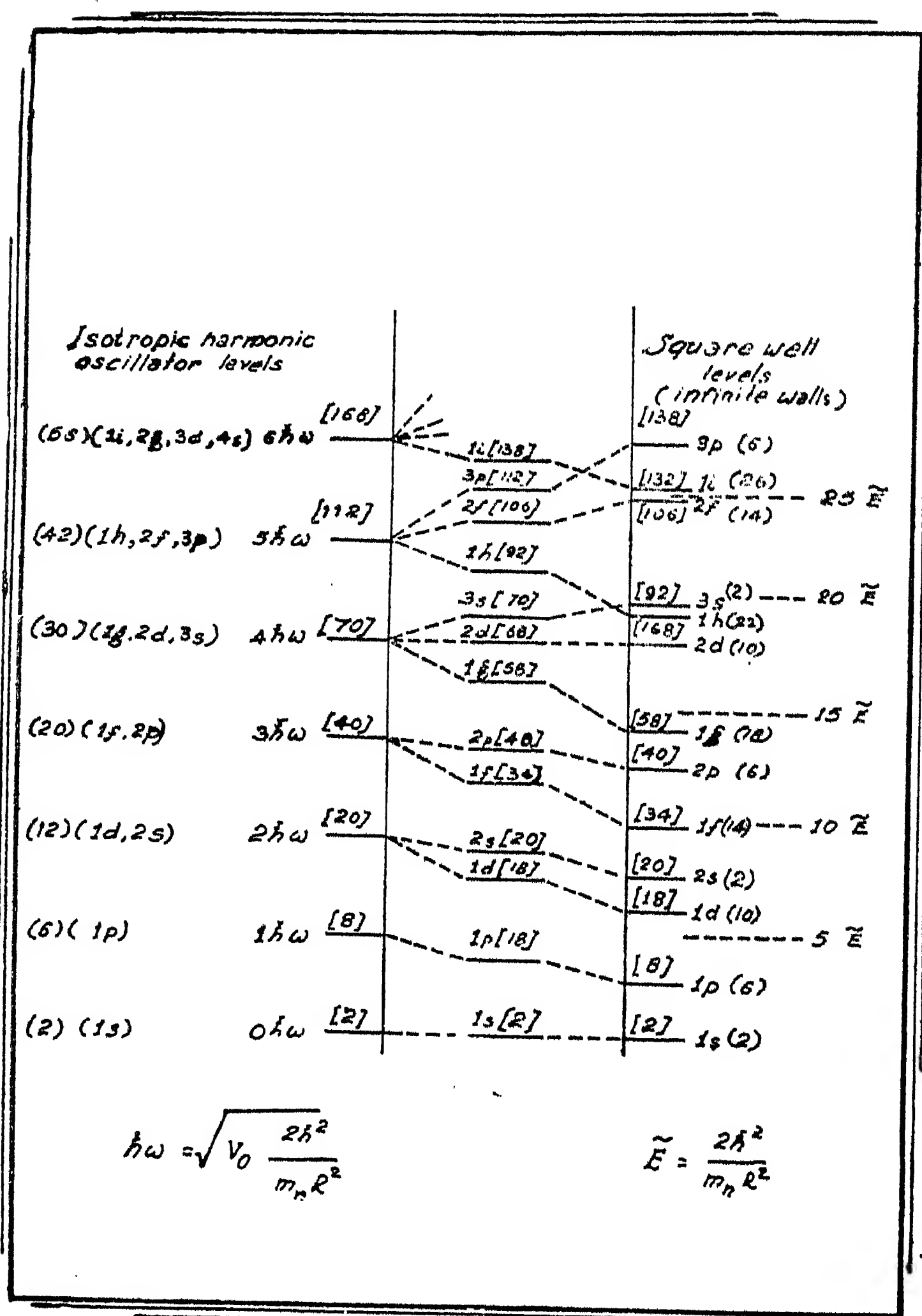


Figure 1.1

highly degenerate because all states with the same $2n + 1$ have the same energy. In the levels of the infinite square well, the degeneracies are removed, the states of angular momentum being shifted to lower energies. The potential intermediate to these two extremes is perhaps the realistic picture of the same.

A level spacing may be expected like the one arbitrarily interpolated in the center of the level schemes of the two extreme cases. Although the spacing is different, the order of levels can be found to be the same except for those in the range of levels $1h$, $3s$ and $1i$, $3p$ states. The pairs in an intermediate potential as expected are close in energies.

This model presents a description of a nuclear state by stating the number of nucleons contained in each of the orbits. There exist for each value of l , $2l + 1$ degenerate sub-states which correspond to different orientations of the angular momentum. Then each of these substates, in accordance with Exclusion Principle, can contain at most two nucleons of each type so that a d-level may be said to be capable of containing ten protons and ten neutrons. Consider the interpolated level scheme given in figure 1.1. The number of protons or neutrons which each level can hold as well as the total number in the same level and all the lower ones can be noted. The level scheme could be realistic or not can be judged by considering one or two examples such as C^{13} and Rb^{87} . The former contains 6 protons and 7 neutrons. It should have 2 protons and 2 neutrons in the $1s$ level and 4 protons as well as 5 neutrons in $1p$ level. In the same way if Rb^{87} with 37 protons and 50 neutrons be considered, all levels below the $2p$ will be completely filled. This level itself could contain its full complement of 6 neutrons. Moreover it would also have 3 protons with 10 neutrons in the $1g$ level.

Besides giving the level assignments, $1s$, $1p$, $1d$, $1f$... $2s$, $2p$, $2d$, $2f$. . . , the fig. 1.1 marks off the degeneracy of the level by brackets () and the total occupation number of all lower levels upto and inclusive the one under consideration by square brackets []. In the level scheme deduced on the basis of this single particle model, large gaps in the spacing of energy levels are observable. These gaps mark off the discontinuities corresponding to nuclear properties such as binding energy, capture cross-section and angular momentum which mark the magic number 2, 8, 20, 40, 70. These numbers, however, do not concur and coincide with the magic numbers experimentally observed save the first three 2, 8 and 20. The shell model is likely to produce energy gaps at the correct magic numbers by

assuming a spin orbit force in addition to the static potential already dealt with given by

$$V_{ls} = - V(r) \mathbf{l} \cdot \mathbf{s} = - V(r) (\mathbf{r} \times \mathbf{p}) \cdot \mathbf{s} \quad \dots (1.3)$$

the vectors referring to the orbital angular momentum, spin, position and momentum of the individual nucleon. This potential which was introduced as a hypothesis to explain the magic number was proved with considerable evidence to be real even though thoroughly satisfactory theoretical explanation of its presence is not so far provided. Its strength suggests that it is due to neither electromagnetic phenomenon nor small relativistic effects but due to nuclear force. The tensor force does not seem to account for it completely.

In the case of spin orbit force the states of different l do not get mixed inasmuch as l commutes with $\mathbf{l} \cdot \mathbf{s}$. The orientation of l is not a constant of motion and only has a fixed Z -component. Any given l state involves two j -values, $j = l \pm \frac{1}{2}$. In the expression (1.3.), the factor $\mathbf{l} \cdot \mathbf{s}$ is indicative of the energy of a nucleon with parallel orbital and spin angular momenta, being different from that of a nucleon for which these vectors are antiparallel. If $V(r)$ be positive, it is attractive when l and s are parallel. Therefore each level of the sequence is split into two, corresponding to two j -values, with $j = l \pm \frac{1}{2}$ being the lower level. The magnitude of splitting increases with l and can be proportional to $(2l+1)$ provided the recoil dependence of $\mathbf{l} \cdot \mathbf{s}$ potential is smooth. The l -dependence of the splitting may to some extent be modified near the nuclear boundary where it may largely be concentrated.

A further clarification can be had if the level sequence as modified by strong spin-orbit coupling, the schematic diagram of which is given in Fig. 1.2, is observed. The left hand side of this figure represents the (n, l) levels of the middle part of the previous one (fig. 1.1). These (n, l) levels are connected by dotted lines with the resultant pairs of (n, l, j) levels. A group of levels, separated from other levels by relatively wide energy gaps is characterised as the nuclear shell. It may be noted from Fig. 1.2 that an S-state ($l=0$) is not split whereas splitting of P-states is small. Thus the gaps persist at 2 and 8. It

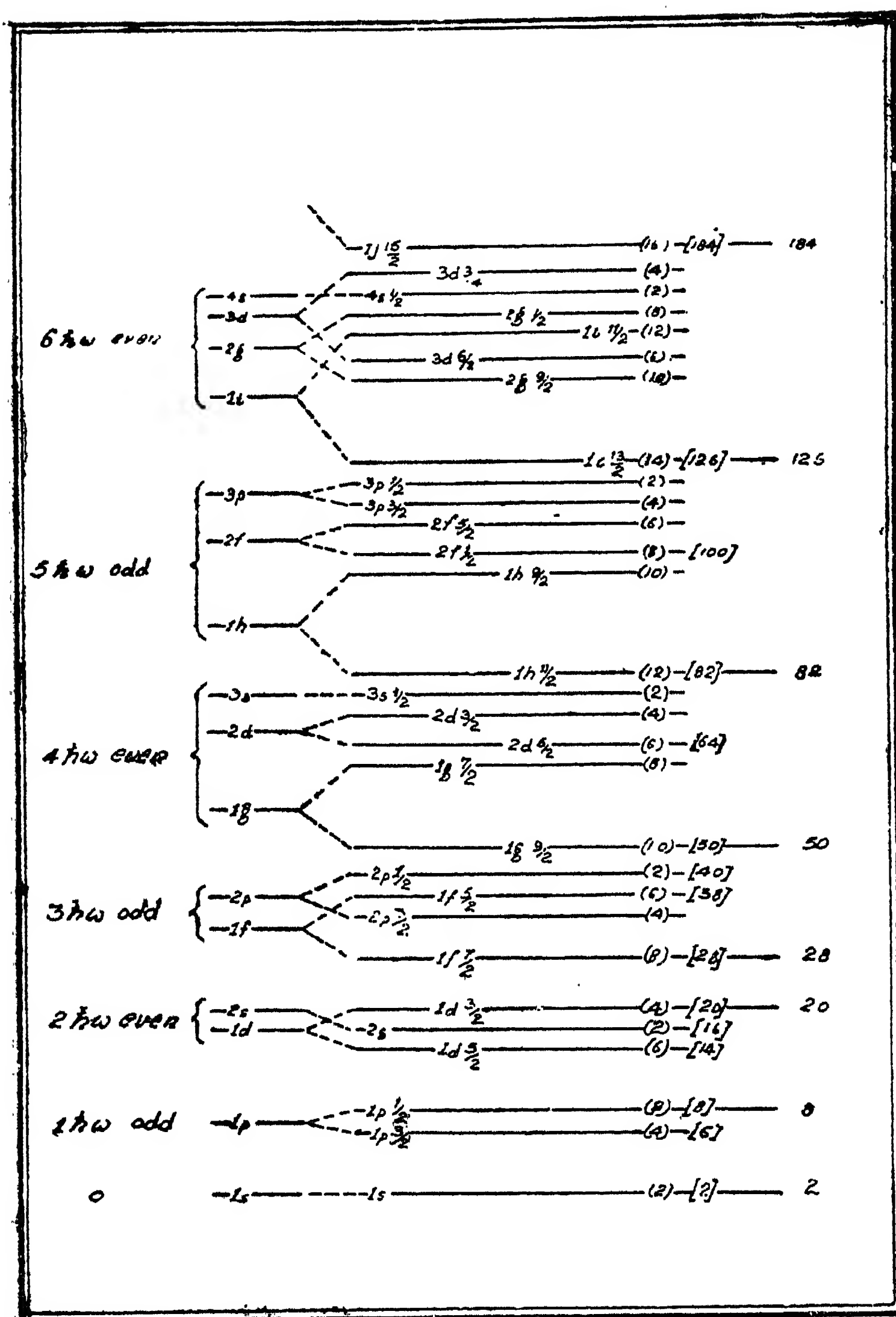


Figure 1.2

may be noted that $l + \frac{1}{2}$ level is lower than the $l - \frac{1}{2}$ level such as $1d_{5/2}$ which is lower and $1d_{3/2}$ which is higher than

the original $1d$ state (Fig. 1.2). The nuclear shells are not the same as the oscillator shells due to spin orbit coupling. It may be noted that the grouping in the three lowest levels ($1s$) ($1p$) ($1d, 2s$) is not appreciably changed so that the lower shell numbers 2, 8, 20 remain unaffected by the spin orbit coupling. The $1f_{7/2}$ level of the next oscillator level group is appreciably depressed and the $1f_{5/2}$ is raised by the $1s$ splitting there by bringing about fair isolation from other levels which is indicative of the shell closure at the nucleon number 28. The splitting of the $1g$ level from the higher oscillator group brought down close to the preceding level group gives rise to a distinct gap at nucleon number 50. It is clear that the spin-orbit coupling plays a dominant role in the level arrangement. A similar situation arises bringing forth similar features with $1h_{11/2}$ and $1i_{13/2}$ levels at the upper edges of the crowded groupings where occur the nucleon numbers 82 and 126 respectively.

The level order within nuclear shell 51 to 82 arises mainly from the oscillator level group which corresponds to $4\hbar\omega$ of even parity except for the level of highest spin $1g_{9/2}$ which has been filled at 50. It may be noted that the level of the highest spin $1h_{11/2}$ of the level order within the nuclear shell of odd parity is brought down into this nuclear shell. Thus all levels of this nuclear shell save that of $1h_{11/2}$ have even parity being brought down from the immediate oscillator level group corresponding to $5\hbar\omega$ of odd group. The levels of this group are all of odd parity except for the level $1i$ which is brought down from the even group being already filled with at 126, the magic number. The level order within each shell will depend on the properties of the potential assumed. The Fig. 1.2 represents the nuclear level scheme with spin-orbit coupling mainly for protons. The pattern for neutrons is nearly the same for the protons up to $N=50$. From then on certain changes take place. The low neutron angular momenta are less disfavoured than the low proton angular momenta. The nuclear level scheme with spin-orbit coupling differs from that for protons. The $2d_{5/2}$ level is below the $1g_{7/2}$ level, the $3s_{1/2}$ level is below the $2d_{5/2}$ level, the $2f_{7/2}$ is very close to the $1h_{9/2}$ level and also $3p_{3/2}$ level is probably below the $2f_{5/2}$ level.

1.3 The Phase of Single Particle Model as a Simple Form of Shell Model

The extreme single particle model constitutes the simplest form of shell model in which the nucleons are assumed to be in the ground state having dynamically paired motions. Many of the nuclear properties, are therefore due to the last unpaired nucleon. Also it is assumed that the neutron and proton states fill independently which amounts that the state into which a proton goes is independent of the number of neutrons or the state into which a neutron goes is independent of the number of protons in the nucleus. In this type, every even-even nucleus has a zero spin and every odd-A nucleus has the angular momentum of the unpaired particle. The angular momentum of odd-odd nucleus cannot be predicted in as much as there is no basis to find which of the various possible resultants of the j -vectors of the unpaired particles has lowest energy. This model in its simple procedure does not at all take into account the nucleon correlation and thereby is unable to cover the actual state of the nucleus. It is probably appropriate to consider only the nucleon in closed shells to form an inert core and also take into account the internucleon forces between the unfilled orbits. It can be shown that this model in its realistic approach predicts that the lowest state of an even number of neutrons or protons in the same j -shell is $J=0$ and the lowest state of an odd number is $J=j$. The experimentally determined Values of J for odd-A type nuclei enable the level order determination of the shell model. The data of the experimentally obtained values of spin of odd-A type of nuclei together with the expected configurations of nucleons are tabulated and given in table (1-1). A comparison of the data contained therein with those computed on the basis of this model will be useful. If there is agreement the validity of the model within the assumed precincts is ascertained whereas discrepancies are cognized in order to explore the cause of such a behaviour.

From the table it can be seen that filling of the successive states of level sequence in general occurs in order until N or Z is reached 33 except for the ninth proton being in the $1d_{5/2}$ - level

in F^{17} and in $2S_{1/2}$ - state in F^{19} . These two isotopes are immediate neighbours and their behaviour exhibits the occurrence of small variations from nucleus to nucleus in the region of effective central potential. It is interesting to note in particular that the neutron number appears to have some effect on the mode of filling the proton shells thereby making it evident that the assumed independence of neutron and proton shells is only approximate. There are also other cases calling for attention. The nucleon number 11 (Ne^{21} , Na^{23}) in which three nucleons in $d_{5/2}$ - orbitals are assumed to complete their angular momenta to a total J of $3/2$. Similarly the number 25 (Ti^{47} , Mn^{55}), where $f_{7/2}$ nucleons are coupled to $J = 5/2$. A departure is observable from the level expected to be filled at nucleon number 33. The $p_{3/2}$ shell is filled by the 32nd neutron or proton. Consequently the 33rd neutron or proton is expected to be found in $1f_{5/2}$ - state. Nevertheless the nuclei with 33 neutrons or protons have spin $3/2$ as many nuclei with 35 or 37 nucleons of one kind have. Similarly the $2d_{5/2}$ state is expected to be filled at $N = 56$ although the spins of $5/2$ are found for nuclei with 57, 59 and 61 neutrons thereby indicating that the filling in pairs occurs in the states of higher angular momentum. It is a well known fact that the 51st to 58 neutrons are in $(2d_{5/2})^6 (1g_{7/2})^2$ configuration. With the addition of the 59th neutron, the same does not enter as the third $g_{7/2}$ neutron. On the other hand, the closed $d_{5/2}$ shell is split up resulting in the configuration $(2d_{5/2})^5 (1g_{7/2})^4$.

The root cause of such an unexpected behaviour may be traced to pairing energy which has a definite bearing on the observed level order. The principle involved is that the configuration in which pairs of nucleons have zero angular momentum are energetically favoured and that the binding energy of nucleons for every such pair is increased by an amount p above the sum of all individual particle binding energies. It is found that the magnitude of this pairing energy increases with the angular momentum j of the single particle level in which pairing occurs. The pairing energy ascertains precisely the manner in which closely neighbouring levels are filled when nucleons are added one after the other to the nucleus. The overall picture of pairing energy can be had by considering a nucleus with an even number of neutrons with binding energy

$B(N)$. With an addition of two neutrons $B(N+2) - B(N)$ turns out to be always greater than $2 \{ B(N+1) - B(N) \}$. Hence there is an extra-pairing energy between two neutrons in the same shell over and above the individual binding energy. The same situation arises with protons.

The extreme phase of single particle model is unable to account for the source of this extra-pairing energy. However, in the case of internucleon interaction pairing energy increases with the j -value of the particles. The $1g_{7/2}$ level is above the $2d_{5/2}$ level. Even so the binding energy may be gained by the break up of the $d_{5/2}$ -shell with the consequent addition of the extra pair in the $g_{7/2}$ state. If allowance be made for the principle of filling by pairs of the $1f_{5/2}$ shells, the $g_{9/2}$ proton shell, the $1g_{7/2}$ neutron shell, the $1h_{11/2}$ shells and the $1i_{13/2}$ neutron shell, as well as for the very closeness of energy of certain levels such as the $1g_{7/2}$ and $2d_{5/2}$ proton states, the single particle model accounts in a very natural way for the J values of almost all the odd nuclei except of course for a few isolated cases such as Se^{79} and Hg^{207} , the exceptional J values all occurring for $151 \leq A \leq 180$ and $N \geq 140$. In the regions of these mass values the nuclei are considerably deformed from sphericity so that the level sequence of the spherical potential cannot be expected to be applied in this region. There are deformed nuclei also in other part of the periodic table for which the present level scheme is not applicable. However, the prediction of large number of experimental J values indicates the validity of this model. Moreover, it predicts also the magnetic moments on the Schmidt lines. Thus the agreement is such that the underlying significance of the model is confirmed. The large measure of agreement while confirming the importance of the extreme single particle model calls for the necessity of its refinements. The refinements involve that an allowance for interaction between particles in unfilled shells be made while maintaining the filled shells as forming an inert core which amounts to the use of the single particle model.

1.4 The Modified Single Particle Model

The modified single particle model is a refinement of the extreme single particle model. In this model all the low energy

properties are attributed to its loose particles outside the closed shells. The interactions of these loose particles with one another are assumed so small that they do not perturb them appreciably from the single particle orbits described by the quantum numbers (n, l, j) . Certain types of topics for which correlations in the motion of the particles are important cannot be dealt with and in so far this model cannot be stated to be any improvement over that of the extreme single particle.

The degeneracy resulting from the different states formed by k particles with the same n, l, j met within the extreme single particle model can be removed by assuming the interaction to be just strong enough as to remove the degeneracies but not so strong compared with the spin orbit force as to cause the cessation of j as a good quantum number for each nucleon. This assumption which is so to speak a part of the single particle model may be relaxed while maintaining the fundamentals in view by adopting configuration mixture consisting of a substitution of the wave functions of two or three (n, l, j) -states for that of the particle.

The mentioned procedure does not imply that the interaction potential v_{ij} between particles in a shell is the same as that in the two particle system. A considerable part of the interaction is accounted for in the potential of the single particle model. This potential represents the average effect on one nucleon of all the other nucleons. The force which removes the degeneracy of shell model states is referred to as a remanent effective interaction, the exact form of which, although difficult to ascertain may be computed by employing simple analytic forms.

In the case of the treatment of light nuclei $A \leq 50$, the total i -spin of a nucleus which plays an important role may be defined as

$$T = \sum_{i=1}^A t_i \quad \dots (1.4)$$

t_i being the i -spin vector. A quantum number may be said to be good if it is invariant for a nuclear state. Since the isobaric quantum number $T_3 = \frac{1}{2} (Z - N)$, it is a good quantum number. T^2 also is a constant of motion to the extent to which the inter-nucleon forces are charge independent. The two nucleon

nuclear force to a good approximation can possibly be charge independent. However, the Coulomb force is not so. In the case of light nuclei it is reasonable to treat $T^2 = T(T+1)$ as a good quantum number of the system since the Coulomb energy is a part of the whole. As the value Z^2/A increases, the approximation, however, no longer holds good. The line that demarcates the two regions is characterized by the increase of the excess of neutrons so that T ceases to be a good quantum number and the different single particle shells thereby being filled by neutrons and protons.

The wave function, according to exclusion principle, is required to be antisymmetric under interchange of all coordinates of any pair of nucleons. If the shell be one only of protons, then the wave function of the angular momentum must be antisymmetric in all pairs. On the other hand if the shell be one containing both neutrons and protons and T a good quantum number, the function of the angular momentum must be symmetric or antisymmetric for an exchange of a given pair depending on the symmetry of the i -spin function for the same exchange. From some of the restrictions mentioned it can be seen that the construction of an antisymmetric state with J -odd from two particles with the same value of j is not possible since the appropriate wave functions for two (n, l, j) particles combining to yield angular momentum (J, M) , expressed in the Clebsch-Gordan symbols is

$$U_n(r_1) U_n(r_2) \sum (jjmM-m | JM) X_j^m(1) X_j^{M-m}(2) \dots (1.5)$$

If the particles (1) and (2) are interchanged the equation (1.5) becomes

$$U_n(r_1) U_n(r_2) \sum (jjmM-m | JM) X_j^m(2) X_j^{M-m}(1)$$

which on changing the summation index by introducing $m' = M-m$ and employing the property,

$$(jjm_1m_2 | JM) = (-)^{j_1+j_2-J} (j_2j_1m_2m_1 | JM)$$

becomes

$$\begin{aligned} & U_n(r_1) U_n(r_2) \sum_m (jj M-m'/m' | JM) X_j^{m'}(1) X_j^{M-m'}(2) \\ &= U_n(r_1) U_n(r_2) \sum (-)^{2j-J} (jjm'M-m' | JM) X_j^m(1) X_j^{M-m}(2) \dots (1.6) \end{aligned}$$

The j is half an odd integer. Therefore (1.6) and (1.5) differ by the factor $(-)^{J+1}$. Hence antisymmetric functions may be observed to exist only for even J .

The three particle configurations $(n, l, j)^3$ are subjected to the same restrictions as the two particle configurations. The former $(n, l, j)^3$ can be shown to have no state $J=\frac{1}{2}$ antisymmetric for the exchange of all pairs of nucleons. The possible antisymmetric states for k -particles for values $j \leq 11/2$ are given in table 1.2. The possible states for $2j+1-k$ particles which amount to k -holes in the shell are the same as for k -particles. For fixing the state of a system of k -nucleons, in addition to the quantum numbers J and T are required the quantum numbers known as the seniority S and the reduced, i -spin t , the former being the number of particles left in the shell after the removal of any such antisymmetric pair for which $T=1, J=0$ and the latter being the i -spin of these remaining nucleons. Two i -spin states for each particle being available, utmost two particles only can be in a given space state with the same (n, l, j, m) . In the same way there may be space symmetric pairs which will be automatically antisymmetric in i -spin. Moreover, there may be particles, the space states of which are different from those of all other particles and which form space exchange antisymmetric pairs with other nucleons. These pairs are i -spin symmetric $T=1$. Again some out of these space antisymmetric pairs may have resultant $J=0$.

Let the interaction potential v_{ij} between two particles be small compared with the single particle forces. Then the contribution of this potential to the energy is the expectation value given by $(\sum_{i < j=1}^k v_{ij})$ for the single particle in view. If v_{ij} is a

central non-exchange force of the type of Wigner potential, a closed expression for the energy contribution can possibly be derived in terms of S, T, s, t and certain integrals of the radial function (7, 8). However, since the nuclear potential contains spin exchange forces a general expression for $\sum v_{ij}$ may be obtained by assuming that (1) the force range is $\ll$ than the radius of the nucleus and (2) all particles in the shell have same charge. Only under these conditions the singlet (antisymmetric) states contribute so that the potential given by

$$V = -V_0 \left[N(r) + B(r) P_\sigma + M(r) P_r + M(r) P_M \right]$$

the various functions of r being arbitrary, becomes

$$\langle \Sigma v_{ij} \rangle = (W + M - H - B) V_0 (2j + 1) f_{nl} \left[K - \frac{(2j + 3 - S)}{2(j + 1)} S \right] \quad \dots (1.7).$$

$$\text{where } V_0 = 4\pi \int V_0(r) r^2 dr,$$

and

$$f_{nl} = \frac{1}{4} \int \left[r U_{nl}(r) \right]^4 \frac{dr}{4\pi r^2}$$

where the term $(W + M - H - B) V_0(r)$ represents the attractive 'S (n, p) system. The seniority quantum number S is less than $2j$ and f_{nl} is positive. Therefore, lowest energy is obtained with the smallest value of S . In the case of even number of nucleons k , the smallest possible value of S is zero and for odd k , the lowest value of S is one. Wherever S is zero, all nucleons are coupled in pairs to $J = 0$ and all nucleons but one are so coupled. The ground state spin of an odd number of like nucleons with the same n, l, j is $J = j$.

In predicting the spin of a nucleus by employing the preceding results it will be necessary to assume further that the effective interparticle forces between nucleons in different l and l' are weak coupled with those between two particles in the same orbit on the basis that nuclear forces are of short range and that there is overlap of the l - and l - wave functions which amounts to the statement that angular momentum coupling within a shell, whether filled or not is not upset by the internucleon interaction with other shells. Therefore, in the case of even Z and odd N nuclei, the protons couple to $J=0$ and the neutrons to $J=j_n$, the j - value of the neutron shell which is filling so that the resultant angular momentum of the whole nucleus is $J=j_n$. Similarly in the case of a nucleus of an odd Z and even N , $J=j_p$ whereas in the case of even nucleus J remains zero. The simple shell model is capable of predicting the correct values of angular momentum even though the specific internucleon forces in unfilled shells are completely ignored.

The ambiguity inherent in the extreme single particle model for odd-odd nuclei persists even here. A neutron shell

$(n_n \ l_n \ j_n)^{k_n}$ with the total angular momentum j_n and proton shell $(n_p \ l_p \ j_p)^{k_p}$ with angular momenta coupled to j_p , with the value of the resultant angular momentum J can be any where from $(j_n - j_p)$ to $(j_n + j_p)$. The ensuing degeneracy of the J -values may be removed by considering interaction energy between neutrons and protons in the two unfilled shells on the assumption that forces calculated for the individual shells are not changed. Now to compute the energy of the mentioned configuration, let $|(j_n)^{k_n} (j_p)^{k_p}, J\rangle$ be the wave function for the

states which the k_n neutrons are coupled to j_n , the k_p protons are coupled to give j_p and the two resulting states then being coupled so that their vector sum $J_n + J_p$ has a magnitude J , the interaction energy between the individual neutrons and protons in the unfilled shells may be expressed as

$$\sum_{i=j}^{K_n} \sum_{j=1}^{K_p} \langle (j_n)^{k_n} (j_p)^{k_p}, J | v_{ij} | (j_n)^{k_n} (j_p)^{k_p}, J \rangle \dots (1.8)$$

The expression (1.8) may be reduced to a simple general form if short range forces are assumed so that the potential is given

as a delta function; $V_o (\vec{r}_{ij}) = V_o \delta (\vec{r}_{ij})$. Within this limit, space exchange loses its significance and Heisenberg and Bartlett forces are spin exchange forces whereas Majorana and Wigner forces have no exchange character so that the potential v_{ij} may be expressed as

$$v_{ij} = - (1 - \mathcal{L} + \mathcal{L} \vec{\sigma}_i \cdot \vec{\sigma}_j) V_o \delta (\vec{r}_{ij}) \dots (1.9)$$

where $\mathcal{L} = \frac{1}{2} (H + B) = \frac{1}{2} (1 - W - M)$

Incorporating the above mentioned assumptions, the energy expressed by (1.8) is calculated and found to be^(9,10)

$$\begin{aligned} E (j_n)^{k_n} (j_p)^{k_p}, J &= E_o (K_n, K_p) + E \sigma \\ &= \lambda_n \lambda_p E_o (1, 1) + (K_n K_p - \lambda_n \lambda_p) (1 - \mathcal{L}) \\ &\quad F_o (l_n l_p) + E \sigma \dots (1.10) \end{aligned}$$

Where

$$\lambda_n = \frac{2 j_n + 1 - 2 K_n}{2 j_n - 1}$$

$$F_o(l_n, l_p) = \frac{V_o}{4\pi} \int (r u l_n)^2 (r u l_p)^2 \frac{1}{r^2} d r,$$

$$E_o(1, 1) = \frac{1}{2} (1 - \lambda) F_o(l_n, l_p) (2j_n + 1) (2j_p + 1)$$

$$\times (j_n j_p \frac{1}{2} - \frac{1}{2} | J O)^2 (2J + 1)^{-1}$$

$$\times \left[\frac{\left\{ (j_n + \frac{1}{2}) + (-)^{j_n + j_p + J} (j_p + \frac{1}{2}) \right\}^2 + 1}{J(J + 1)} \right]$$

and

$$E_\sigma = \frac{1}{2} \lambda F_o(l_n, l_p) (2j_n + 1) (2j_p + 1) (j_n j_p \frac{1}{2} - \frac{1}{2} | J O)^2 (2J + 1)^{-1}$$

$$\times \left[\frac{\left\{ (j_n + \frac{1}{2}) + (-)^{j_n + j_p + J} (j_p + \frac{1}{2}) \right\}^2}{J(J + 1)} - \left\{ 1 + 2(-)^{l_n l_p J} \right\} \right]$$

The energy E_σ denotes the one resulting from forces involving exchange and it is independent of the number of nucleons in the shells.

In the case of a shell with only a single proton and neutron energy is given by

$$E \{ (j_n)^1, (j_p)^1, J \} = E_o(1, 1) + E_\sigma$$

and it is evidently a complicated function of J . The potential expressed by equation (1. 9) implies that the triplet interaction is V_o whereas the singlet is $(1 - 4\alpha)V_o$. If the net spin exchange force be attractive α is positive and the triplet interaction is stronger than the singlet interaction. This situation is true for two nucleons which are free so that the orientation of the two nucleons is expected to favour the triplet spin state. Thus σ_n and σ_p are parallel. If $j_p - l_p$ and $j_n - l_n$, each of which being $\pm \frac{1}{2}$, are of opposite sign, parallel σ 's are obtained for opposite orientation of j_n and j_p . Hence $J = |j_n - j_p|$, whereas if $j_p - l_p$ and $j_n - l_n$, be of the same sign, the inclination towards parallel spins renders $J = (j_n + j_p)$ the state of lowest energy

The state with $J=(j_n - j_p)$ however, is a triplet state containing a singlet component. The preceding reasoning holds if the preference for the triplet state is distinctly clear so that a is not too small. It has been shown^(9, 11) that for even forces of finite range, the state $J=|j_n - j_p|$ is energetically well below the other possible states if $j_p - l_p$ and $j_n - l_n$ are of opposite sign provided $a > 1/10$. If these quantities are of the same sign, the energy of the state $J=j_n + j_p$ is lowered and if a is adequately large the state $j_n - j_p$ may be crossed by it. The spacing between the states being small, the expected order may be altered by the perturbations concomitant with their smallness.

The results of the preceding discussions regarding the jj coupling of odd-odd nuclei are generalized and briefly expressed in the form of simple coupling rules by Nordheim⁽¹²⁾ and modified by Brennan and Bernstein⁽¹³⁾. Let $J_p - l_p + j_n - l_n = N$ be termed Nordheim number. Then the spin J of an odd nucleus may be written as

$$N = 0, J = |j_n - j_p|;$$

$$N = \pm 1, J \text{ is either } |j_n - j_p| \text{ or } j_n + j_p$$

being respectively termed the strong and weak rules which hold for k_n and $k_p > 1$. Then the total angular momenta of the configuration $j_n^{k_n}$ and $j_p^{k_p}$ may be written as

$$N = 0, J = |j_n - j_p|;$$

$$N = \pm 1, J \text{ is either } |j_n - j_p| \text{ or } j_n + j_p.$$

In exceptional cases J_n and $J_p \neq j_n$ and j_p . These empirically derived rules have been theoretically established⁽¹⁴⁾ on the basis of spin dependent residual interaction and also the nature of the effects which can bring about the inversion in the expected order of the closely spaced states in the event of the applicability of the weak rule is clarified. A close survey of the observed spins and parities will indicate when two orbits are close together in energy, an odd nucleon in an odd nucleus is not always in the same orbit in the odd-odd nucleus as in the corresponding odd-A nucleus.

1.5 Configuration Admixture

The particles, as already mentioned, are considered to move in (n, l, j) orbits and in such a situation all states of k -particles in the same configuration will be degenerate. The consequent degeneracy may be removed by introducing interactions and calculating the energy splitting between different states by the theory of the first order perturbation. The energy effective of the inter-section is assumed to be not sufficiently strong to appreciably change the wave functions of the states. It is, however, required to investigate its validity by considering a nucleus with the extreme single particle wave function ψ_0 (JM) which corresponds to a configuration of the form given by

$$(n_1, l_1, l_1 + \frac{1}{2})^{k_1} (n_1, l_1, l_1 - \frac{1}{2})^{K_1} (n_2, l_2, l_2 + \frac{1}{2})^{k_1} (n_2, l_2, l_2 - \frac{1}{2})^{K_2} \dots \quad (1.11)$$

The majority of the shells, $k_i = 2l_i + 2$ and the majority of the $K_i = 2l_i$ are filled, only one or two of the shells being un-

filled. The different modes of coupling the vectors $\vec{j}_i$ in the unfilled shells lead to various states of different J . In the absence of nucleon-nucleon interaction these different states are degenerate with energy E_0 . Consider the state ψ_0 (JM) and on introducing internucleon potential v_{ij} , it will alter its energy to a first order by an amount $(\psi_0 | \sum_{i < j} v_{ij} | \psi_0)$ so that

the degeneracy is removed. The wave function, however is simultaneously changed by the interaction and can by the theory of first order perturbation may be represented as

$$\psi_{JM} = \psi_0(JM) + \sum_p \mathcal{L}_p \psi_p(JM) + \sum_q \mathcal{L}_q \psi_q(JM) \quad \dots (1.12)$$

where a_p and a_q are given by

$$\mathcal{L}_p = \frac{(\psi_0 | \sum_{i < j} v_{ij} | \psi_p)}{E_0 - E_p}$$

$$\mathcal{L}_q = \frac{(\psi_0 \epsilon \mid \sum_{i < j} v_{ij} \mid \psi_q)}{E_0 - E_q} \quad \dots (1.13)$$

p and q being single particle states with unperturbed energies E_p , E_q and the summation being inclusive of all those states of the same J and M for which $\mathcal{L}_p$ and $\mathcal{L}_q$ are non zero. Then obviously an admixed state p or q can differ from an unperturbed state ψ_0 by having at most two particles in different states. The states p are distinguished from the states q in that the nucleons in the state have the same n - and l -values as those in ψ_0 . Hence the configuration of the state ψ_p can be expressed as

$$\begin{aligned} & (n_1, l_1, l_1 + \tfrac{1}{2})^{k_1 - a_1} (n_1, l_1, l_1 - \tfrac{1}{2})^{K_1 + a_1} \\ & \times (n_2, l_2, l_2 + \tfrac{1}{2})^{k_2 - a_2} (n_2, l_2, l_2 - \tfrac{1}{2})^{K_2 + a_2} \quad \dots (1.14) \end{aligned}$$

with $a_i = -2, -1, 0, 1$ or 2 and $\sum_i |a_i| = 1$ or 2

A state ψ_q can have a configuration which may be written as

$$\begin{aligned} & (n_1, l_1, l_1 + \tfrac{1}{2})^{k_1 + b_1} (n_1, l_1, l_1 - \tfrac{1}{2})^{K_1 + \beta_1} \\ & \times (n_2, l_2, l_2 + \tfrac{1}{2})^{k_2 + b_2} (n_2, l_2, l_2 - \tfrac{1}{2})^{K_2 + \beta_2} \quad \dots (1.15) \end{aligned}$$

with b_i and $\beta_i = -2, -1, 0, 1$ or 2 , $\sum_i (|b_i| + |\beta_i|) = 2$ or 4 and $b_i + \beta_i \neq 0$ for at least one value of i . This ensures that in a state q at least one nucleon has changed its n and l or n or l .

Even if p -states alone are admixed, each nucleon could be assigned a specific l -value, in which case j would no longer be a good quantum number. However, for q states, neither one of the two, l and j , is a constant of motion. In both types of admixture excitation occurs to unfilled level nucleons initially from both filled and unfilled levels. The coefficients $\mathcal{L}_p$ and $\mathcal{L}_q$ measure the amount of admixture. It is required that all the $|\mathcal{L}|^2 \ll 1$ in order to hold the first order perturbation and quite convincingly to ignore configuration mixing. Only a few states are appreciably mixed in the wave function. These few states are those the unperturbed energies of which, E_p or E_q

Nucleus	N	Z	J	μ	Q barns	Configuration of neutrons				
						$2d_{5/2}$	$1g_{7/2}$	$3s_{1/2}$	$2d_{3/2}$	$1h_{11/2}$
Zr ⁹¹	51	40	5/2	-1.30285		1				
Mo ⁹⁵	53	42	5/2	-0.9133		3				
Mo ⁹⁷	55	42	5/2	-0.9325		5				
Ru ⁹⁹	55	44	5/2	-0.6315		5				
Ru ¹⁰¹	57	44	5/2	-0.6915		5	2			
Pd ¹⁰⁵	59	46	5/2	-0.6155		5	4			
Pd ¹⁰⁹	61	48	5/2	-0.828615		5	6			
Cd ¹¹¹	63	48	1/2	-0.62213		6	8	1		
Sn ¹¹⁵	65	50	1/2	-0.91781		6	8	1		
Sn ¹¹⁷	67	50	1/2	-0.99983		6	8	1	0	2
Sn ¹¹⁹	69	50	1/2	-1.04621		6	8	1	0	4
Te ¹²³	71	52	1/2	-0.73585		6	8	1	0	6
Te ¹²⁵	73	52	1/2	-0.88715		6	8	1	0	8
Xe ¹²⁹	75	54	1/2	-0.77686		6	8	1	0	8
Xe ¹³¹	77	54	3/2	+0.69066	-0.120	6	8	2	1	10
Ba ¹³⁵	79	56	3/2	+0.83718		6	8	2	1	12
Ba ¹³⁷	81	56	3/2	+0.93654		6	8	2	3	12
82										

						$2f_{7/2}$	$1h_{9/2}$	$3p_{3/2}$	$2f_{5/2}$	$3p_{1/2}$	$1i_{13/2}$
Nd ¹⁴³	83	60	7/2	-1.085	+0.05206	1					
Nd ¹⁴⁵	85	60	7/2	-0.69(10)	+0.2084	3					
Sm ¹⁴⁷	85	62	7/2	-0.76(8)	-0.2084	3					
Sm ¹⁴⁹	87	62	7/2	-0.64	+0.0601(1)						
5											

119 to 126 Region of non-spherical nuclei

Pt ¹⁹⁵	117	78	1/2	+0.60602		8	10	4	6	1	6
Hg ¹⁹⁷	117	80	1/2	+0.524056		8	10	4	6	1	6
Hg ¹⁹⁹	119	80	1/2	+0.502702(1)		8	10	4	6	1	8
Hg ²⁰¹	121	80	3/2	-0.556701(3)							
				+0.505		8	10	3	6	0	12
Pb ²⁰⁵	123	82	5/2?			8	10	3	5	0	14
Pb ²⁰⁷	125	82	1/2			8	10	4	6	1	14

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Nucleus	N	Z	J	μ	Q barns	Configuration of neutrons			
						$2g_{9/2}$			
Pb ²⁰⁹	127	82	9/2?			1			
Po ²¹¹	127	84	9/2?			1			
Pb ²¹¹	129	82	9/2?						
130			Region of non-spherical nuclei						
						$1p_{1/2}$	$1p_{3/2}$	$1p_{1/2}$	
H	1	0	1/2	+2.792777		1			
H ³	1	2	1/2	-2.97385		1			
Li ⁷	3	4	3/2	+3.25628		2	1		
B ¹¹	5	6	3/2	+2.68857	+0.036	2	3		
N ¹⁵	7	8	1/2	-0.28309		2	4	1	
8									
						$1d_{5/2}$	$2s_{1/2}$	$1d_{3/2}$	
F ¹⁷	9	8	5/2			1			
F ¹⁹	9	10	1/2	+2.6287			1		
Na ²³	11	12	3/2	+2.21751	+0.11	(3)			
Al ²⁷	13	14	5/2	+3.64140	+0.15	(5)		Non-spherical Nuclei	
P ³¹	15	16	1/2	+1.13166		(6)	(1)		
Cd ³⁵	17	18	3/2	+0.82183	-0.080	6	2	1	
Cd ³⁷	17	19	3/2	+0.68409	-0.063	6	2	1	
K ³⁹	19	20	3/2	+0.39140	+0.092	6	2	3	
K ⁴¹	19	22	3/2	+0.21483	+0.11	6	2	3	

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Nucleus	Z	N	J	μ	Q barns	Configuration of protons
1f _{7/2}						
Sc ⁴⁶	21	24	7/2	+4.75626	−0.22	1
V ⁵⁷	23	28	7/2	+5.143	+0.20	3
Mn ⁵³	25	28	7/2	+5.046		5
Deformed						
Mn ⁵⁵	25	30	5/2	+3.4678	+0.35	5
Co ⁵⁷	25	30	7/2	+4.585		7
Co ⁵⁹	27	32	7/2	+4.5835		7

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2p _{3/2} 1f _{5/2} 2p _{1/2} 1g _{7/2}						
Cu ⁶³	29	34	3/2	+2.2261	+0.163	1
Cu ⁶⁵	29	36	3/2	+2.3849	+0.153	1
Ga ⁶⁹	31	38	3/2	+2.02602	+0.191	3
Ga ⁷¹	31	40	3/2	+2.516161	+0.121	3
As ⁷⁵	33	42	3/2	+1.4390	+0.32	3 2
Br ⁷⁹	35	44	3/2	+2.1056	−0.221	3 4
Br ⁸¹	35	46	3/2	+2.2696	−0.282	3 4
Rb ⁸¹	37	44	3/2	+2.052		3 6
Rb ⁸³	37	46	5/2	+1.422		4 5
Rb ⁸⁵	37	48	5/2	+1.35267		4 5
Rb ⁸⁷	37	50	3/2	+2.7505		3 6
Y ⁸⁹	39	50	1/2	−0.137316		4 6 1
Nb ⁹³	41	52	9/2	+6.1671	−0.21	4 6 2 1
Tc ⁹⁹	41	56	9/2	+5.6806	+0.34	4 6 2 3
Rh ¹⁰³	45	58	1/2	−0.0883		4 6 1 6
Ag ¹⁰⁷	47	60	1/2	−0.113548		4 6 1 8
Ag ¹⁰⁹	47	62	1/2	−0.130538		4 6 1 8
Ag ¹¹¹	47	64	1/2	−0.1461		4 6 1 8
In ¹⁰⁹	49	60	9/2	+5.546	+1.20	4 6 2 9
In ¹¹³	49	64	9/2	+5.5233	+1.143	4 6 2 9
In ¹¹⁵	49	66	9/2	+5.5351	+1.165	4 6 2 9

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Nucleus	Z	N	J	μ	Q barns	Configuration of protons				
						$1g_{9/2}$	$2d_{5/2}$	$1h_{11/2}$	$2d_{3/2}$	$3s_{1/2}$
Sb ¹²¹	51	70	5/2	+3.3590	-0.591		1			
Sb ¹²³	51	72	7/2	+2.5466	-0.68	1				
I ¹²⁷	53	74	5/2	+2.8091	-0.785	2	1			
I ¹²⁹	53	76	7/2	+2.6173	-0.55	3				
I ¹³¹	53	78	7/2	+2.7381	-0.401	3				
Cs ¹³¹	55	76	5/2	+3.534		4	1			
Cs ¹³³	55	78	7/2	+2.5789	-0.0036	5				
Cs ¹³⁵	55	80	7/2	+2.7290	+0.049	5				
Cs ¹³⁷	55	82	7/2	+2.8382	+0.50	5				
La ¹³⁹	57	82	7/2	+2.7781	+2.231	7				
Pr ¹⁴¹	59	82	5/2	+4.01	-0.0589	8	1			
Pm ¹⁴⁷	61	86	7/2	+2.73	+0.9535	8	3			
Eu ¹⁵¹	63	88	5/2	+3.4632	+0.95	8	5	Deformed		
Eu ¹⁵³	63	90	5/2	+1.529	+2.42	8	5			
	64 to 78					Region of non-spherical nuclei				
Au ¹⁹⁷	79	118			+0.56	8	6	12	3	1
Tl ²⁰³	81	122	1/2	+1.61169		8	6	12	4	1
Tl ²⁰⁵	81	124	1/2	+1.62754		8	6	12	4	1

[illegible]

TABLE 1.2

Possible Total Spins J for Various Configurations $(j)^k$

$j = 3/2$	$j = 5/2$	$j = 7/2$
$k = 1: 3/2$ $= 2: 0, 2$	$k = 1: 5/2$ $= 2: 0, 2, 4$ $= 3: 3/2, 5/2, 9/2$	$k = 1: 7/2$ $= 2: 0, 2, 4, 6$ $= 3: 3/2, 5/2, 7/2, 9/2, 11/2, 15/2$ $= 4: 0, 2 \text{ (twice)}, 4 \text{ (twice)}, 5, 6, 8$
$j = 9/2$		
$k = 1: 9/2$ $= 2: 0, 2, 4, 6, 8$ $= 3: 3/2, 5/2, 7/2, 9/2 \text{ (twice)}, 11/2, 15/2, 17/2, 21/2$ $= 4: 0 \text{ (twice)}, 2 \text{ (twice)}, 3, 4 \text{ (3 times)}, 5, 6 \text{ (3 times)}, 7, 8, 9, 10, 12$ $= 5: 1/2, 3/2, 5/2 \text{ (twice)}, 7/2 \text{ (twice)}, 9/2 \text{ (3 times)}, 11/2 \text{ (twice)}, 13/2 \text{ (twice)}, 15/2 \text{ (twice)}, 17/2 \text{ (twice)}, 19/2, 21/2, 25/2$		
$j = 11/2$		
$k = 1: 11/2$ $= 2: 0, 2, 4, 6, 8, 10$ $= 3: 3/2, 5/2, 7/2, 9/2, \text{ (twice)}, 13/2, 15/2 \text{ (twice)}, 17/2, 19/2, 21/2, 23/2, 27/2$ $= 4: 0 \text{ (twice)}, 2 \text{ (3 times)}, 10 \text{ (3 times)}, 11, 12 \text{ (twice)}, 13, 14, 16$ $= 5: 1/2, 3/2 \text{ (twice)}, 5/2 \text{ (3 times)}, 7/2 \text{ (4 times)}, 9/2 \text{ (4 times)}, 11/2 \text{ (5 times)}, 13/2 \text{ (4 times)}, 15/2 \text{ (5 times)}, 17/2 \text{ (4 times)}, 19/2 \text{ (4 times)}, 21/2 \text{ (3 times)}, 23/2 \text{ (3 times)}, 25/2 \text{ (twice)}, 27/2 \text{ (twice)}, 29/2, 31/2, 35/2$ $= 6: 0 \text{ (3 times)}, 2 \text{ (4 times)}, 3 \text{ (3 times)}, 4 \text{ (6 times)}, 5 \text{ (3 times)}, 6 \text{ (7 times)}, 7 \text{ (4 times)}, 8 \text{ (6 times)}, 9 \text{ (4 times)}, 10 \text{ (5 times)}, 11 \text{ (twice)}, 12 \text{ (4 times)}, 13 \text{ (twice)}, 14 \text{ (twice)}, 15, 16, 18$		

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Part Two

Independent Particle Model

The independent particle model is based on the assumption that each nucleon moves independent of all other nucleons in a common potential field representing the average effect of all interactions with other nucleons, this field likely being the same for each particle. Hence this model involves a description of a systematic procedure to derive nuclear wave functions which though considered accurate relies partly on the approximate validity of the shell model. The wave function and the energies of the states of a mechanical system require to postulate a complete set of wave functions which is orthogonal. Let these functions be denoted by Φ_i . These wave functions need not be the actual wave function of the system. This procedure is adopted in order to set up the energy matrix which is formed by evaluating $(\Phi_i | H | \Phi_j)$, H being the Hamiltonian of the system. Then the matrix is diagonalized, the diagonal elements being the eigen values E_k of the energy of the actual system and the corresponding eigen vectors a_{ik} giving the actual wave functions Ψ_A given by $\Psi_A = \sum_i a_{ik} \Phi_i$. A proper choice of the set Φ_i on the basis of physical insight is to be made for keeping the calculation within reasonable limits by diagonalization a part of energy matrix for a given Ψ_k there being a few non-zero a_{ik} 's.

This model considers for Φ_i , completely antisymmetrical products of single particle states for any reasonable central potential. The harmonic-oscillator potential may be employed because of its simplicity. A description of the method of developing antisymmetric wave functions for A nucleons which is adequate for this purpose may be outlined by considering two

usually employed sets of basic wave functions Φ_i , expressed in terms of L—S and of j—j coupling schemes. In the L—S coupling scheme the single particle wave function is taken into account as given by

$$\phi (n_l m_l m_s m_t) = u_{n_l}(r) y_{lm_l}(\theta, \phi) \sigma(m_s) \tau(m_t) \quad \dots (2.1)$$

depicting particles of fixed orbital and spin angular momenta l , m_l and $\frac{1}{2}$, m_s and fixed charge denoted by m_t . The terms in the above expression $\sigma(m_s) = \alpha$ or β and $\tau(m_t) = \gamma$ or δ . The total wave functions $\Phi(l^k)$ for k particles of quantum number l describing states of definite L , M_L , S , M_S , T , M_T may be formed by employing the function ϕ . It may be mentioned that only totally antisymmetric wave functions are allowed. The identification of the states cannot be complete unless an extraquantum number λ , specifying the symmetry of Φ be supplemented to the quantum numbers mentioned⁽¹⁾

In the j—j coupling, single particle functions are employed which are given by

$$\begin{aligned} \phi(n l j m m_t) \\ = u_{n_l}(r) \tau(m_t) \sum_{m_l} (l \frac{1}{2} m_l m - m_l | j m) y_{lm_l}(\theta, \phi) \sigma(m - m_l) \dots (2.2) \end{aligned}$$

depicting particles of fixed total angular momentum j , m of definite charge m_t and of a given magnitude l for orbital angular momentum. These particle functions are so combined as to form a total state $\Phi(j^k)$ of a definite J , M , T , M_T , seniority S and reduced i—spin t . Even here only totally antisymmetric states are allowed.

The possible L—S coupled states of the p—shell and the j—j coupled states of the $P_{3/2}$ shell and the $P_{1/2}$ shell are tabulated and given in tables 2.1 and 2.2. The set of total wave functions in either one of these two cases is complete and any given actual nuclear state may be expressed as a series of either $\Phi(l^k L M_L S M_S T M_T [\lambda])$ or $\Phi(j^k J M_S t M_T)$... (2.3)

However, the set that corresponds closely to the actual nuclear states is the one that is worth wishing for arriving at the actual situation. The L is a good quantum number for purely central interparticle forces whereas J is the only constant of motion

among the various sums of angular momentum for pure spin orbit force proportional to $l.s$ although individual values of l and s are good quantum numbers. The actual situation, however, being one of intermediate coupling with both spin orbit and central forces present, either each one of the couplings or both couplings may be employed in calculations according to the method followed.

2.1 Matrix Elements

The evaluation of the matrix elements $\langle \Phi_i | H | \Phi_j \rangle$ may be reduced to that of two-particle matrix elements. In this procedure, the fractional parentage coefficients may be introduced for convenience. Let a wave function for k particles expressed by $\Phi(j^k JMst TM_T)$ be treated as one to be made up of all suitable combinations of states of $k-2$ particles and two particles so that

$$\begin{aligned} \Phi(j^k JMst TM_T) = & \sum_{\phi(k-2)} \sum_{\phi(2)} (\Phi \{ | \phi(k-2), \phi(2) \\ & \times \{ \Phi(j^{k-2} J' s' t' T') \Phi(j^2 J'' s'' t'' T'') \dots \end{aligned} \quad (2.4)$$

$\{ \Phi(j^{k-2} J' T') \Phi(j^2 J'' T'') \}$ being the properly antisymmetrized wave function of a state composed of two groups of particles, of which one consists of $k-2$ particles having total quantum number as J' and T' and the other comprising two particles with (quantum numbers) J'' and T'' . The angular momentum vector is specified by J, M , and a total i -spin T, M_T . The summation in II-4 is carried over all possible states $\phi(k-2)$ and $\phi(2)$. The numerical coefficients $(\Phi \{ | \phi(k-2), \phi(2))$ are known as coefficients of fractional parentage (*cfp*). The $3/2, 5/2, 9/2$ are the only possible j values for configuration $(5/2)^3$. These coefficients are evaluated⁽¹⁾ and the resultant numerical values for the mentioned configuration $(5/2)^3$ expressed in terms of $j-j$ coupling are given in Table 2.3. The coefficients are expressed in terms of $j-j$ coupling. Similar considerations, however, hold good in the case of $L-S$ coupled states. There are again certain coefficients for expressing the total wave functions besides $(k \{ | (k-2), 2) cfp's$ as a combination of states of single particle and those of $k-1$ particles as

$$\Phi(j^k) = \sum_{\phi(k-1)} \sum_{\Phi} (\Phi \{ | \Phi(k-1), \Phi \} \{ \Phi(k-1) \Phi \} \dots \quad (2.5)$$

Φ being a wave function of the k -th particle.

The nuclear Hamiltonian in the case of two body forces can be written as

$$H = \sum_i T_i + \sum_{i < j} v_{ij} \quad \dots \quad (2.6)$$

v_{ij} being the interaction between particles i and j . A matrix element of the form $(\Phi_r | v_{ij} | \Phi_s)$ has the same value inasmuch as Φ 's are antisymmetric in all particles. Therefore

$$(\Phi_r | \sum_{i < j} v_{ij} | \Phi_s) = \frac{1}{2} k(k-1) (\Phi_r | v_{k-1,k} | \Phi_s) \quad \dots \quad (2.7)$$

An expansion of Φ_r and Φ_s , on the basis that different $\Phi(k-2)$ are orthogonal and that $v_{k-1,k}$ depends on the $(k-1)$ and the k th particles may give

$$\begin{aligned} (\Phi_r | v_{k-1,k} | \Phi_s) = & \sum_{\phi(k-2)} \sum_{\phi(2)} \sum_{\phi'(2)} (\Phi_r \{ | \Phi(k-2), \Phi(2) \})^* \\ & \times (\Phi_s \{ | \Phi(k-2), \Phi'(2) \} (\Phi(2) | v_{k-1,k} | \Phi'(2)) \quad \dots \quad (2.8) \end{aligned}$$

Consequently if the matrix elements of v_{12} for all possible two-particle states be known the matrix elements for the interaction energy of k -particles can be found. It is required to have the data of the values of cfp for $(k-2)$ as well as those of two-particle grouping. Similarly the matrix elements of sums of one particle operators such as the kinetic energy or the magnetic moment can be expressed in terms of matrix elements between single particle states and the $(k \{ | k-1, 1) cfp$.

It is possible to determine the energies E_k and the wave functions Ψ_k of the actual states of nucleus and obtain as $\Psi_k = \sum_i a_{ik} \Phi_i$ by evaluating and diagonalizing the matrix $(\Phi_r | H | \Phi_s)$. Also the matrix element between two real (actual) states Ψ_k of any operator O such as electromagnetic moment may be expressed as

$$\langle \Psi_k | O | \Psi_m \rangle = \sum_{\mu, \nu} a_{\mu k}^* a_{\nu m} (\Phi_\mu | O | \Phi_\nu) \quad \dots \quad (2.9)$$

If $O_{km} = (\Psi_k | O | \Psi_m)$, $O'_{\mu\nu} = (\Phi_\mu | O | \Phi_\nu)$ and consider $\mathcal{L}$ and O as matrices, the expression (2.9) can be written as

$$O = \mathcal{L}^H O' \mathcal{L} \quad \dots \quad (2.10)$$

The calculation of O' is easier than that of O directly inasmuch as the structures of Φ are simpler than those of Ψ .

2.2 Fixed Positions of the Centers of the Potential and Mass

The choice of the fundamental single particle wave functions Φ as eigen function of a one body potential amounts to an attribution of a fixed position to the center of potential. But it is the center of mass that is actually likely to be fixed. The view of the fixed center of the potential does not coincide with that of the fixed center of mass. A model may be pictured which leads to independent oscillatory motion of all $3A$ degrees of freedom of the space coordinates of the A nucleons whereas the three coordinates in the mass center are actually fixed. It may be said that $3A - 3$ independent coordinates describe the internal motion. The functions ϕ as functions of the internal coordinates are not all linearly independent.

Consider the separation of the center of mass motion to be possible by assuming the wave functions Φ as products of a function of center of mass coordinates R and a function of internal coordinates. Then the functions Φ having the same R dependence can be chosen. Such of these functions are linearly independent forming a complete set in internal degrees of freedom. The remaining functions being spurious can be ignored. This procedure may be extended to harmonic oscillator wave functions which are usually employed for the basic states. All those wave functions Φ containing center of mass coordinates in $1s$ state can be determined⁽²⁾. These wave functions Φ should be divided by the $1s$ -wave functions of the mass center to obtain the wave functions for the internal motion. Most of the matrix elements of interest are for operators which are independent of the mass center. The adoption of the above procedure may seldom be required. The separation of

the center of mass motion in other single particle potentials for harmonic oscillator with spin-orbit added cannot however, be made uniquely⁽³⁾. Therefore the use of shell model wave functions is restricted for many calculations. Even so an approximate correction can be effected.

2.3. A Survey of the requisites for calculation on the basis of independent particle model

Consider the separation of the center of mass motion to be possible by assuming the wave functions Φ as products of a function of center of mass coordinates R and a function of internal coordinates. Then the Φ having the same R -dependence can be chosen. Such of these functions are linearly independent forming a complete set of internal degree of freedom. The remaining functions being spurious can be ignored. This procedure may be extended to harmonic oscillator wave functions which are usually employed for the basic states. All those wave functions Φ containing center of mass coordinates in the $1s$ -state can be determined⁽²⁾. All these wave functions can be divided by these $1s$ -wave functions of the mass center to obtain the wave functions for the internal motion. Most of the matrix elements of interest are for operators which being independent of the mass center, the adoption of the above procedure may seldom be required. The separation of the center of mass motion in other single particle potentials for harmonic oscillator with spin-orbit added cannot however, be made uniquely. Therefore the use of shell model wave functions is restricted for many calculations. Nevertheless an approximate correction can be effected. The choice of suitable functions Φ , the knowledge of the nuclear Hamiltonian in general and that of the forces in particular are essential requisites in order to determine the nuclear properties on the basis of independent particle model. The potential energy can be employed in order to describe the interaction between the nucleons in nuclei inasmuch as these nucleons are expected to be moving with adequately low energy. The data of the two nucleon scattering may be employed but the best values for independent particle calculation cannot be expected to be observed to act between two free nucleons since the presence of other nucleons

might influence the interaction although the evidence of such many body-forces, if at all, is trivial. For simplicity in calculation a different potential is used. The assumption that the states are not different from those of the actual wave functions does simplify the calculations in diagonalizing the energy matrix in the independent particle model. This model envisages that the approximation of a single particle orbital structure is fairly satisfactory. In some contexts this assumption works well. The various parameters associated with these orbits are those such as energy angular momentum etc. of nucleons. Thus the characteristics of the wave functions are looked upon as the coordinates of the nucleons. The success of the model, however, does not adequately substantiate its picture. If there were certain degrees of freedom of the nucleus satisfying particle like equations of motion, the energy and angular momentum characteristics would be unaltered. This concept may be clarified if the example of motion of a two particle system is considered which from the point of view of classical mechanics may be separated into two quasi-particle motions. The change of the coordinates of the mass center appears as if it is one with the total mass of the system acted on by the interparticle force. The coordinates of these fictitious particles turn out to be a particular type of combinations of the coordinates of the actual particles. The use of nucleon coordinates may lead to a form of coordinate system, some of the coordinates of which satisfy equations of motion of the form as that of the equations of motions of single particles. The quasiparticle concept may lead to an understanding of the shell structure. The characteristics of quasiparticles being not identical with those of the nucleons, the nuclear characteristics such as magnetic moment cannot be computed from the assignment of the nucleon moments to the quasiparticles.

From the foregoing considerations, it seems reasonable to modify the two nucleon potential for application to the independent particle model in a manner that each particle is assumed to be experiencing a potential to $1/s$ where s and l are proportional to its angular momenta with l referring to the origin of coordinates. Even if this force be not present in

nucleon-nucleon interactions, it may arise from interactions of a nucleon with a closed shell. The suggestion that l.s force may be due to tensor coupling is untenable inasmuch as its magnitude is too meagre to produce this effect. The effective l.s force, on the other hand may possibly be due to two nucleon L.S coupling and in many cases the matrix elements of $\sum_i s_i \cdot l_i$ are akin to those of $\sum_{ij} (s_i + s_j) \cdot r_{ij} \times (p_i - p_j)$. Another modification may be effected which involves the central force being taken as one of the exchange type which differs from that of the two nucleon potential such as the Rosenfeld potential ⁽⁴⁾ of the form which though essentially central, exhibits spin and parity dependence given by

$$V = -V_0 [W(r) + B(r)P\sigma + M(r)P_x + H(r)P_n] \dots (2.11)$$

where $W = -0.13$, $M = +0.93$, $H = -0.26$, $B = +0.46$ (2.11a)
The use of extreme form of potential, disregarding small components of W and H gives

$$W = H = 0, M = 0.8, B = 0.2 \dots (2.11b)$$

There is also the Serber exchange which differs from the values of the potentials already dealt with and the values in this case of W , M , H , B are obtained as

$$W = M = 0.5, H = B = 0 \dots (2.11c)$$

This latter form is suggested on the basis of the two-nucleon scattering data. A consideration of Serber force does not yield correct value of the difference between the levels with $T = 0$ and $T = 1$ in odd-odd nuclei such as $Li^{(6)}$ and F^{18} . However, in the calculations so far carried out, the radial dependence of the forces has not been assumed to be monotonically attractive. The results are not very sensitive to the shape even for the variations from Yukawa to Gaussian forms. Nevertheless a repulsive core appreciably may alter the results. Also the quasi particle interaction could not be a two-nucleon force. If the actual nucleon-nucleon potential be insisted, the quasiparticles could not remain as such but become real so that the use of the assumption of the shell model, involving only a small part of the energy matrix in diagonalisation becomes a vague procedure. In most of the calculations using this model, the tensor force is not taken in to account and those

calculations may not be involving the properties sensitive to this force.

2.4 Results of Calculations on the Basis of Independent Particle Model

The energy levels for nuclei with mass numbers $A = 7, 10, 18$ and 19 predicted on the basis of independent particle model may be taken into consideration in order to determine its success as well as to ascertain its limitations. With this object in view only the lowest configuration for function ϕ and the diagonalization of the small matrix are taken into account as a measure of simplifying the calculations of this model. The nucleus with $A=7$ may be viewed as a closed shell $(1s)^4$ and the three particles in the p-shell as containing twelve independent single particle states. The d-shells are not taken into consideration and the holes in the $1s$ are not allowed. Similarly in the nucleus with $A=18$ the states of the form $(1s)^4(1p)^{12}(1d,2s)^2$ are taken into account. In such cases the validity of the results is limited to the composition of the particles in the indicated compositions. The facts of the functions ϕ closely resembling the actual wave functions will be of importance in so far as it simplifies the calculations while its usefulness being dependent on the approximate validity of shell model. In the case of nuclei $16 < A \leq 40$, both the $1d$ and $2s$ model functions are to be treated inasmuch as the functions of these are very close in energy. The actual wave functions turn out to be their linear combinations.

The mentioned approximation using the lowest major configuration yields a state of parity as that of the ground state. The states of opposite parity are obtained by an introduction of some particle excitation so that a probable state arising out of particle excitation of the $1p$ nucleons in the nucleus $A = 18$ is the $(1s)^4 (1p)^{11} (1d, 2s)^3$ which is evidently an odd parity state.

The difference in energy between particle states is controlled by the shell model potential from which the basic functions ϕ of the shell are determined. It may be mentioned that energy spacings between the states of the coupling of different angular

momentum within a shell generally are smaller than the mentioned energy differences so that a number of states of excitations of normal parity can be lower than the first state of non-normal parity.

This situation may be understood by following the calculations⁽⁵⁾ for intermediate coupling in 1p-shell for $A = 7$ and 10. The use of extreme form of the potential given by (2-11b) yields results which depend only on L , K and ξ , L and K being certain matrix elements of the central potential and ξ the measure of strength of the l.s force. L and K are expressed in terms of ξ

$$\int_0^{\infty} V(r) e^{-r^2/b^2} (15b^4 r^2 + 4r^6) dr$$

and
$$\int_0^{\infty} V(r) e^{-r^2/b^2} r^4 dr,$$

where $V(r)$ is the interparticle central potential and b is the parameter of the oscillatory functions used for ϕ . In the case of a range of certain values of potential and the size of nucleus the value of the ratio L/K is estimated and found to be approximately 6. Thus the value of k fixes the energy scale whereas L/K and ξ/K determine the relative energy levels. The diagonalization of 1p-shell matrix yields values in terms of the mentioned parameters for Li^7 and Be^7 as well as for B^{10} and Be^{10} as shown in figure 2.1 (a, b) and 2.2 (a, b, c, d, e, f) respectively. In these figures the abscissa represents ξ/k measuring the relative spin-orbit and central energy configurations which are important for computation. The ordinate is given in units of energy. The L/K depends on the ratio of the nuclear size to range of nuclear forces. The diagonalizations are carried out in units of K , being then left as a parameter to match the experimental energy scale. In the Figure 2.1, (a) represents the dependence of the spectrum on ξ/K and L/K where the curves are for $L/K = 6.8$ and illustrates the ξ -dependence and (b) represents calculated levels for $L/K = 6.8$, $\xi/K = 1.0$, $K = -1.2$ MeV. In this case it can be noted that

Energy units of ϕ

MeV

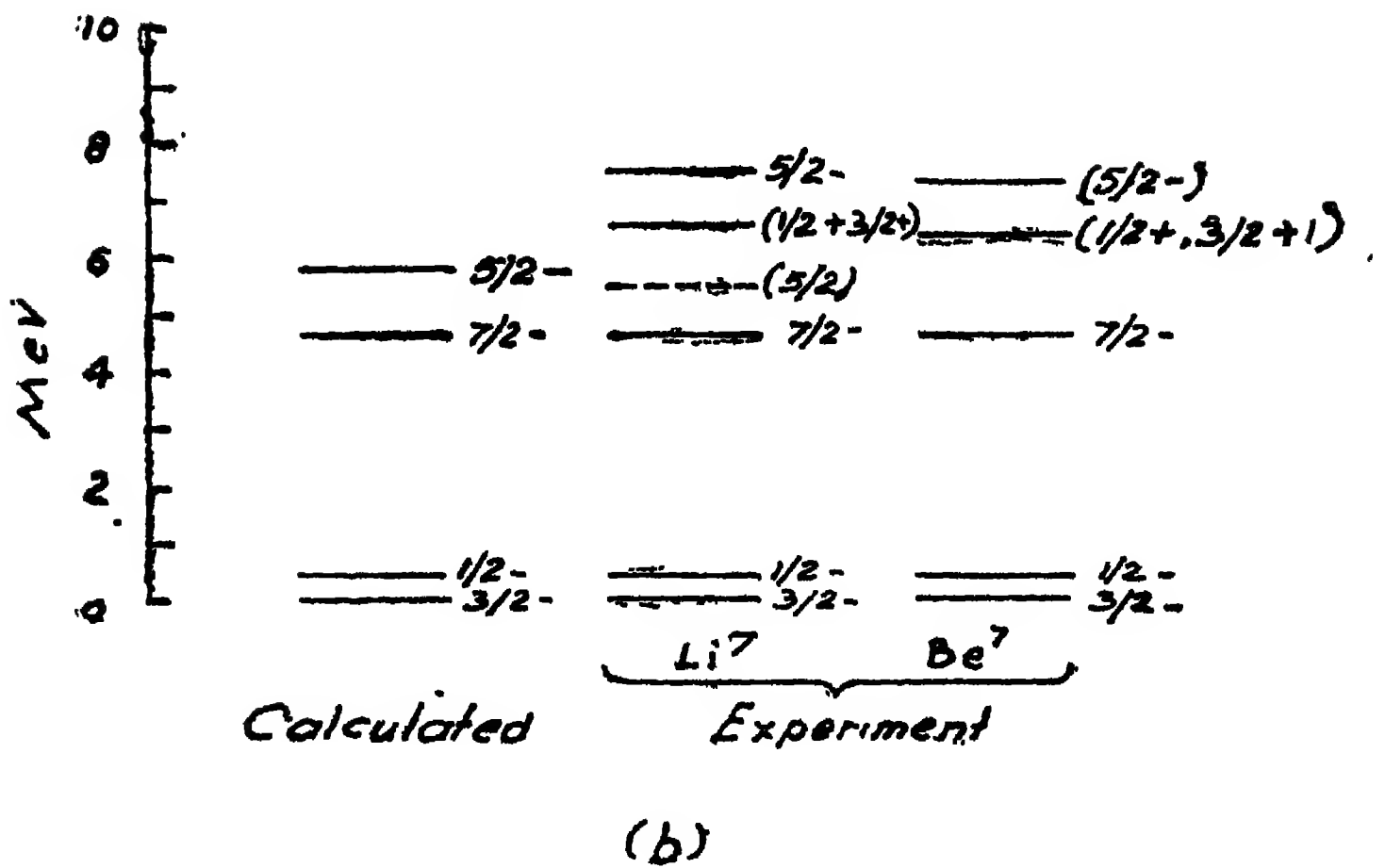
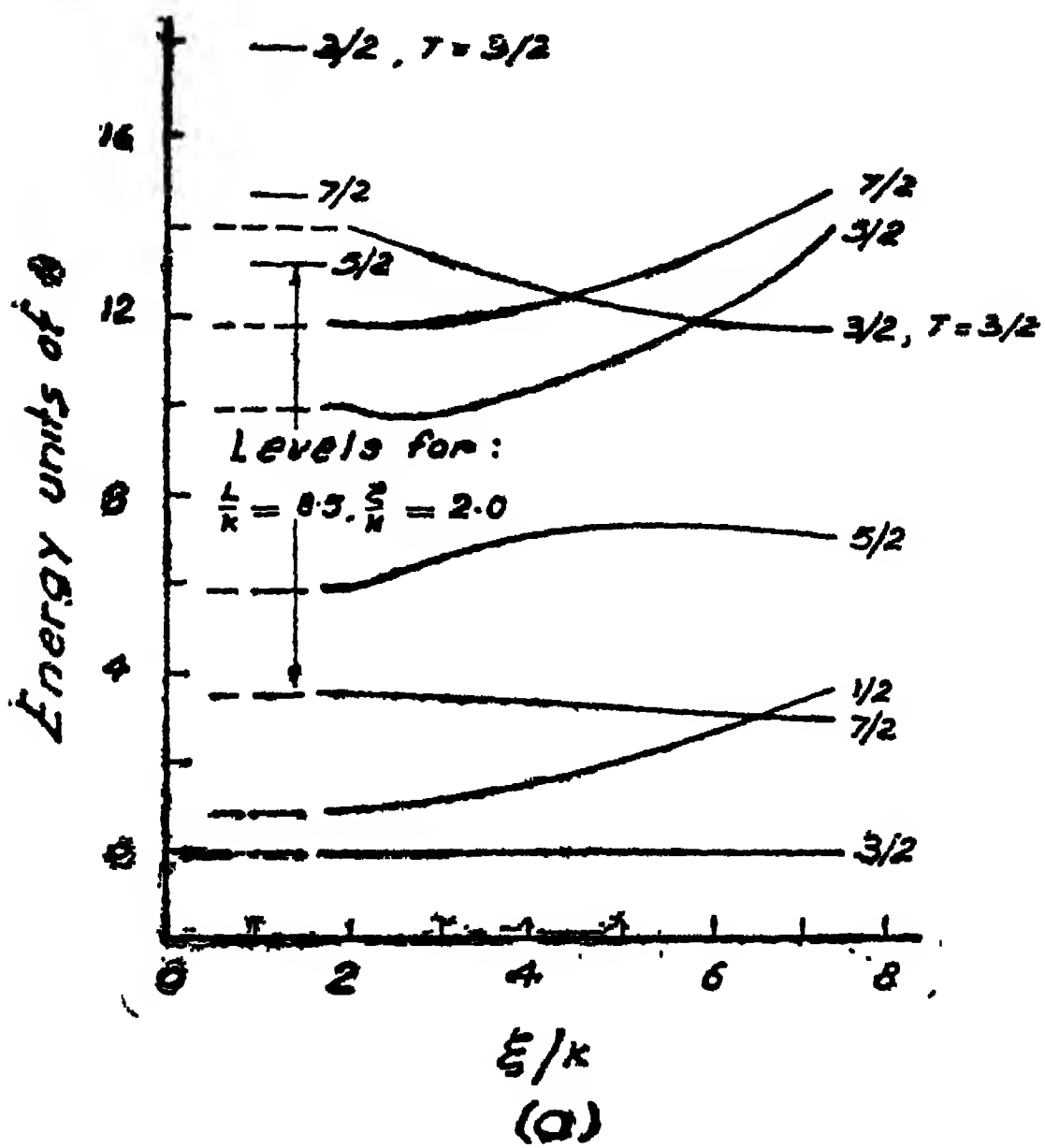


Figure 2.1

the $5/2$ level is in agreement with the calculated $5/2^-$ at 5.7 MeV. Just above this level, a state of opposite parity $1/2^+$ or $3/2^+$ occur which may be considered as the lowest state of particle excitation, the configuration being $(1s)^4 (1p)^2 (1d, 2s)^7$. The next state of normal parity may be expected to be $5/2^-$ as found. The energy, however, is 7.5 MeV which differs from the computed value, 12.5 MeV. The 7.5 MeV state may be said to be resulting from a mixture of mainly the $5/2$ configuration at 12.5 MeV with other higher configurations including particle excitations. It should be so since for a wave function which contains a mixture of various configurations its actual energy is lower than that of the lowest pure configuration provided that these configurations are not mixed into the nuclear states of lower energy as required by the general result of quantum mechanics⁽⁶⁾. From this example, in the case of at least small A it looks as though the nuclear states up to excitation energy of about 5 MeV would be pure configuration but above that energy interconfiguration mixing could be of importance.

The Figure 2.2 shows Kurath's computed results on the levels of the nucleus $A = 10$, of the most complex p-shell type, representing its level-order curves of which (a) indicates levels as a function of ξ/K for fixed $L/K = 6.8$; (b) levels as a function of L/K for fixed $\xi/K = 4$; (c) levels for $\xi/K = 4.75$, $L/K = 6.8$, $K = -0.10$ MeV; (d) levels for $\xi/K = 4.0$, $L/K = 5.8$, $K = -1.13$ MeV; (e) and (f) experimental spectra. It may be mentioned that $A = 10$ are the most complex p-shell nuclei. From this figure it can be seen that there is a slight dependence of the level energies on the ratio L/K determined by the central potential and a more sensitive dependence on the ratio ξ/K . The Be^{10} has only $T = 1$ states since T_z value is 1. The $T = 0$ states are lower than $T = 1$ by about 2 MeV. Such splitting may be observed in some other nuclei which may be connected with the symmetries of the states and the presence of the large attractive spin exchange force of the potential of the Rosenfeld type. The other potential which is of the Serber's type has no adequate spin-exchange interaction to give rise to these splittings as observed by Redlich⁽⁷⁾. The theoretically deduced values with experimental results are in good agreement. The lowest levels, attributed to be resulting from

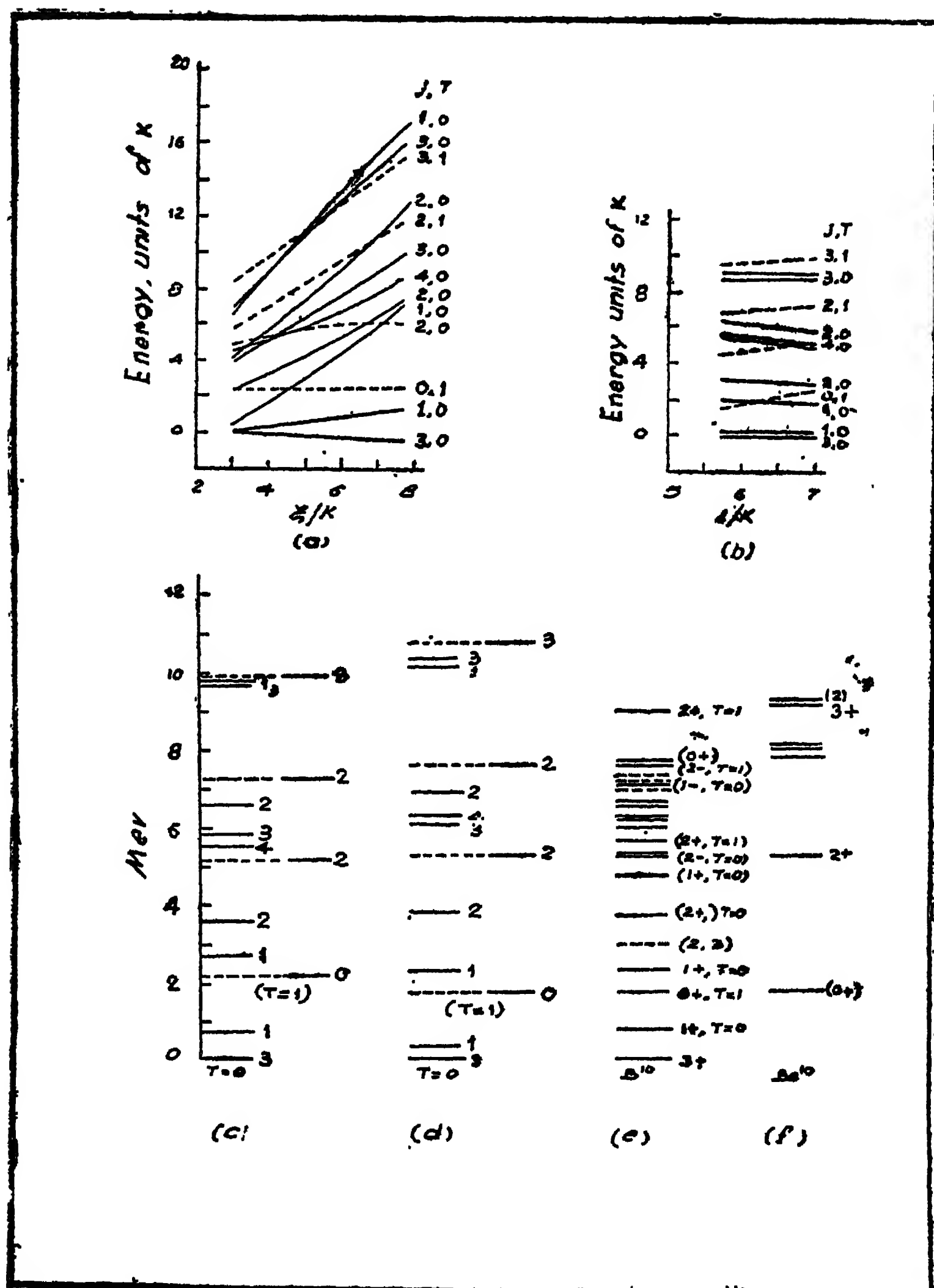


Figure 2.2

particle excitation are actually the 1^+ and 2^- states in the proximity of 5 MeV. The 1^+ state is expected to be a two particle excitation such as $(1s)^4 (1p)^4 (1d, 2s)^2$ on account of its

being of even parity. The 2^- state on the other hand is likely to be the lowest level of $(1s)^4 (1p)^5 (1d, 2s)^1$. The two particle excitation, as already pointed, has slightly lower energy than the one particle state — a fact assumed to be due to a reflection of the finer details of the exchange nature of the forces and symmetries of the two states. This effect is of the same type as that leading to pairing energy.

The calculated levels of all p-shell nuclei elucidate the nature of agreement between the computed values on the basis of individual particle model and the results of experiment. The interesting feature of the outcome of these calculations is that very nearly the same value may be employed throughout the central force parameters L and K whereas the effective single-particle $l.s$ force constant increases with the increase of the shell filling which may be expected in such a case where the spin-orbit force is an approximate representation of diverse complicated spin-dependent interactions with other particles.

In the case of nuclei beyond the configuration $(1s)^4 (1p)^{12}$ of O^{16} the filling of $(1d, 2s)$ - shell is expected. The $1d_{5/2}$ and $2s$ single particle states are close together while the $1d_{3/2}$ level may be expected to be higher. These exceptions may be substantiated in the spectrum of O^{16} shown in Fig. 2.3. The $1d_{3/2}$ level is likely to be higher as is evident in the experimental spectrum O^{17} given in Figure 2.3. The odd parity levels as low as 3 MeV which may be observed therein are caused by either the excitation of the core or lifting up the last neutron to the highest state of the shell model. In this context it may be mentioned that the calculations of normal parity states for $A=18, 19$ etc. cannot be disturbed by these particular simple particle levels inasmuch as the states of opposite parity cannot mix. Nevertheless for energies of some levels the calculated results may turn out to be slightly erroneous by ignoring the probable reasonably low states of two particle excitation and normal parity so that a certain measure of uncertainty may prevail in the calculation of $(1s, 2s)$ - shell due to a number of parameters as much as ten which include the eight radial integrals, $l.s$ constant ξ and the splitting between the single particle d - and s -levels. In the case of the choice of special particle force,

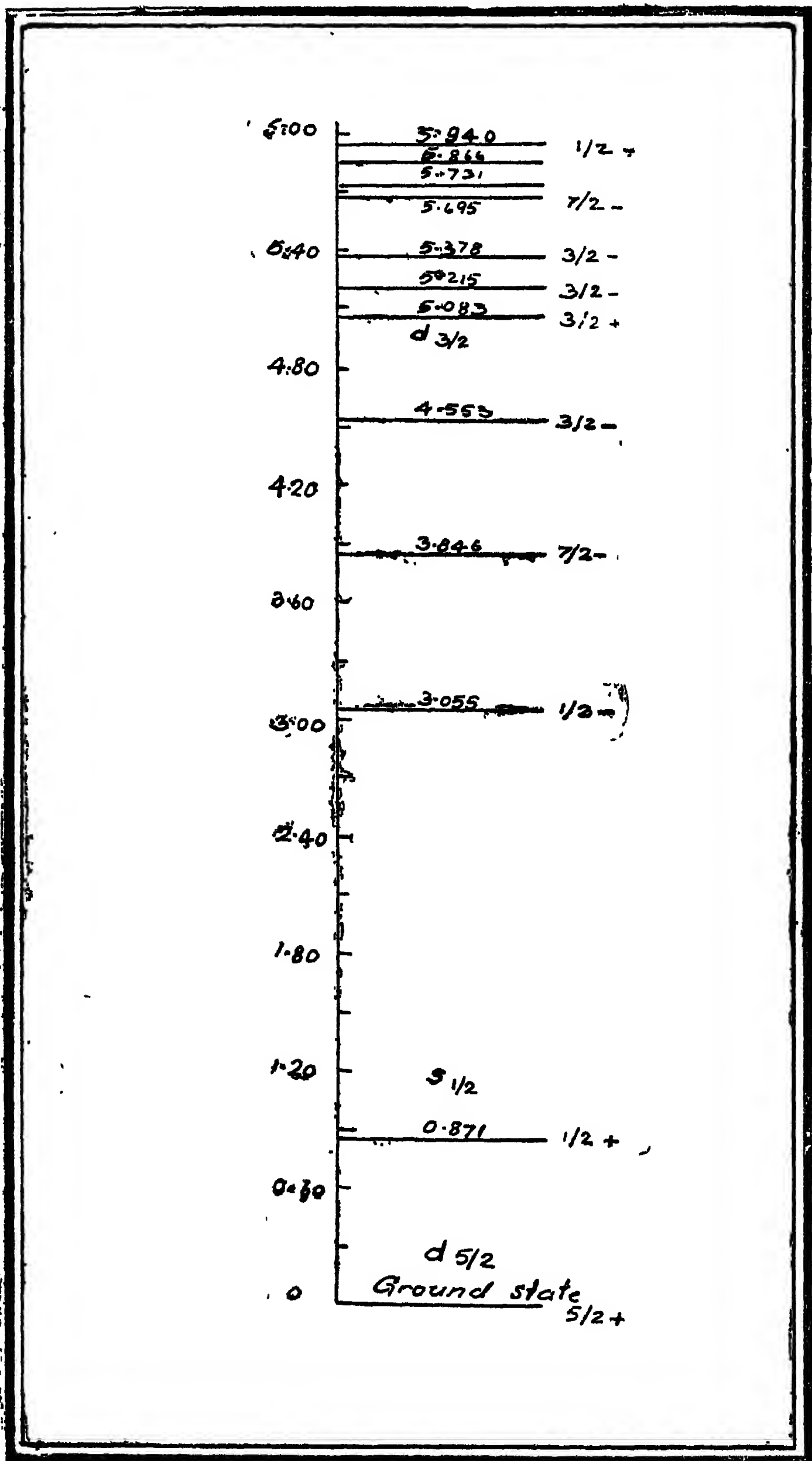


Figure 2.3

the particles are not all independent. The experimental results for $A=17, 18, 19$ involve matching a considerable number of levels. Hence the good agreement is not indicative of the validity of the model. The calculated value¹ for $A=18, 19$ shown in Figs. 2.4 and 2.5 indicate the dependence on one parameter, the strength of the central potential with other parameters fixed. Further the relative position of the $T=0$ and $T=1$ levels is sensitive to the exchange character of the force.

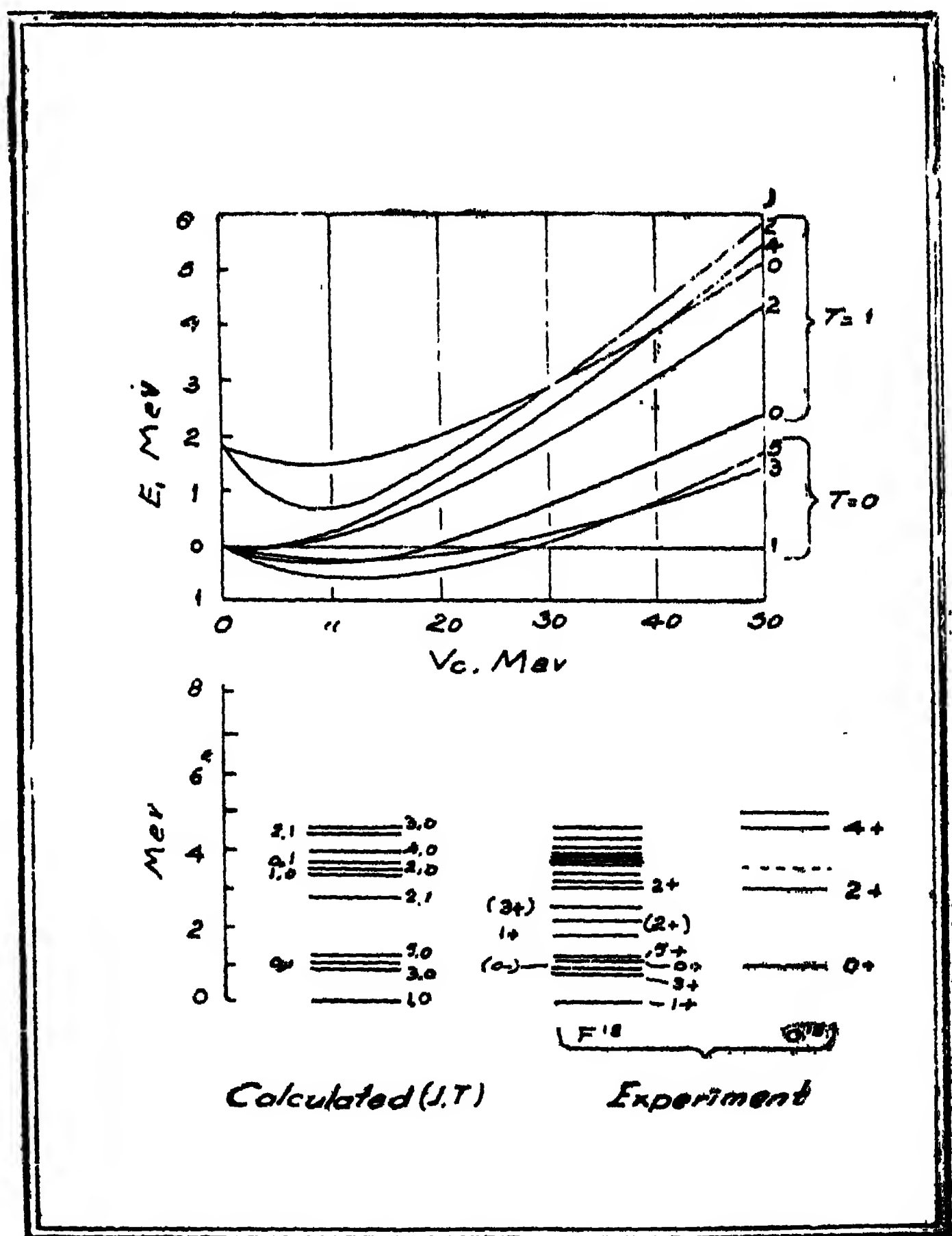


Figure 2.4

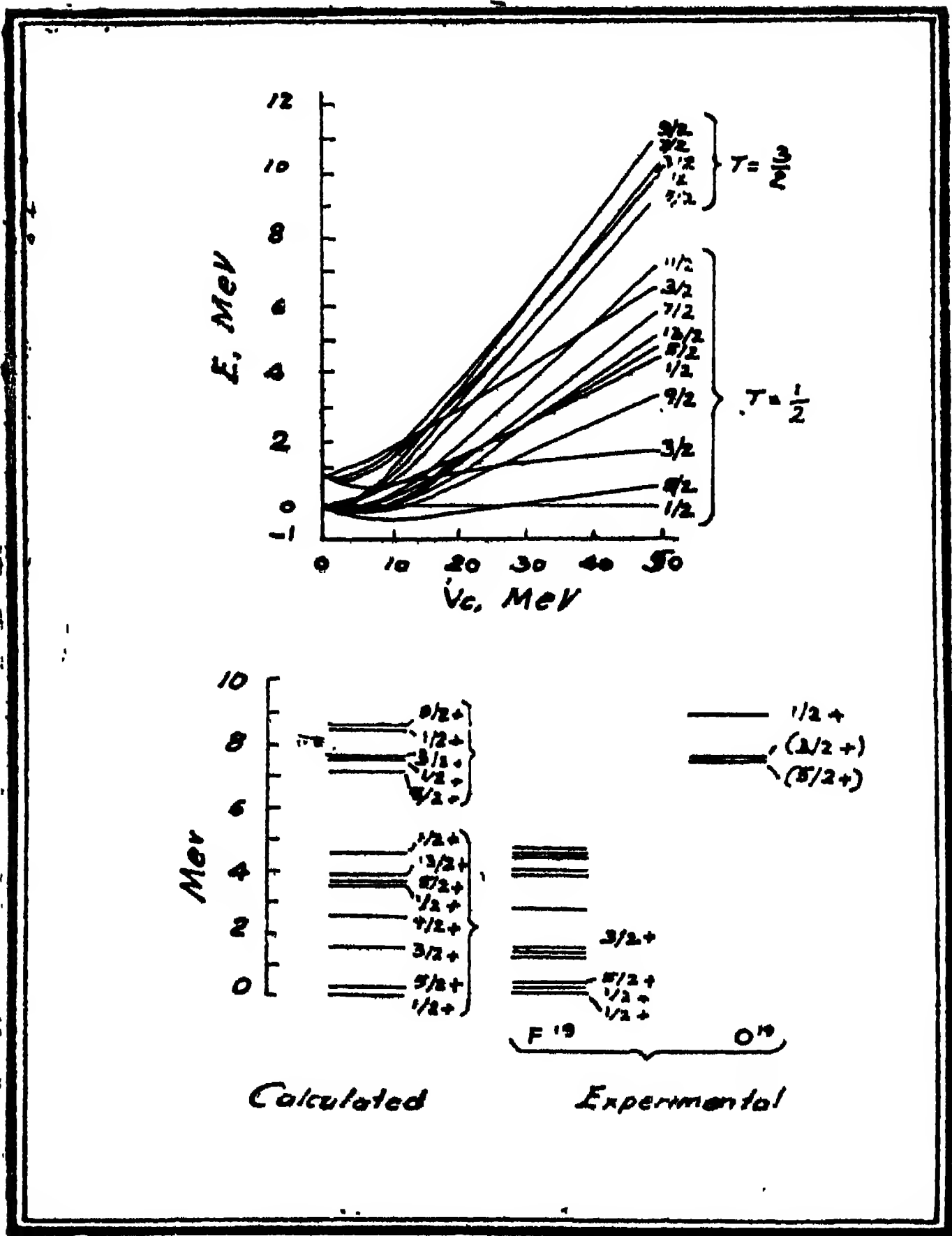


Figure 2.5

The results for $A=19$ represent in the case of F^{19} the very low state of opposite parity at 112 keV. The results are fairly impressive even though obtained from the model formulated on a theoretical basis with a certain measure of uncertainty (Fig. 2.6). The results dealt with so far with a consideration of the data¹¹ of the percentage analysis into jj coupled configurations of the wave functions for A^{19} emphasize the importance of the shell model coupling states by indicating how the wave functions of

a number of single states of the nuclei with $A=19$ are constituted as linear combinations of basic pure jj configurations while

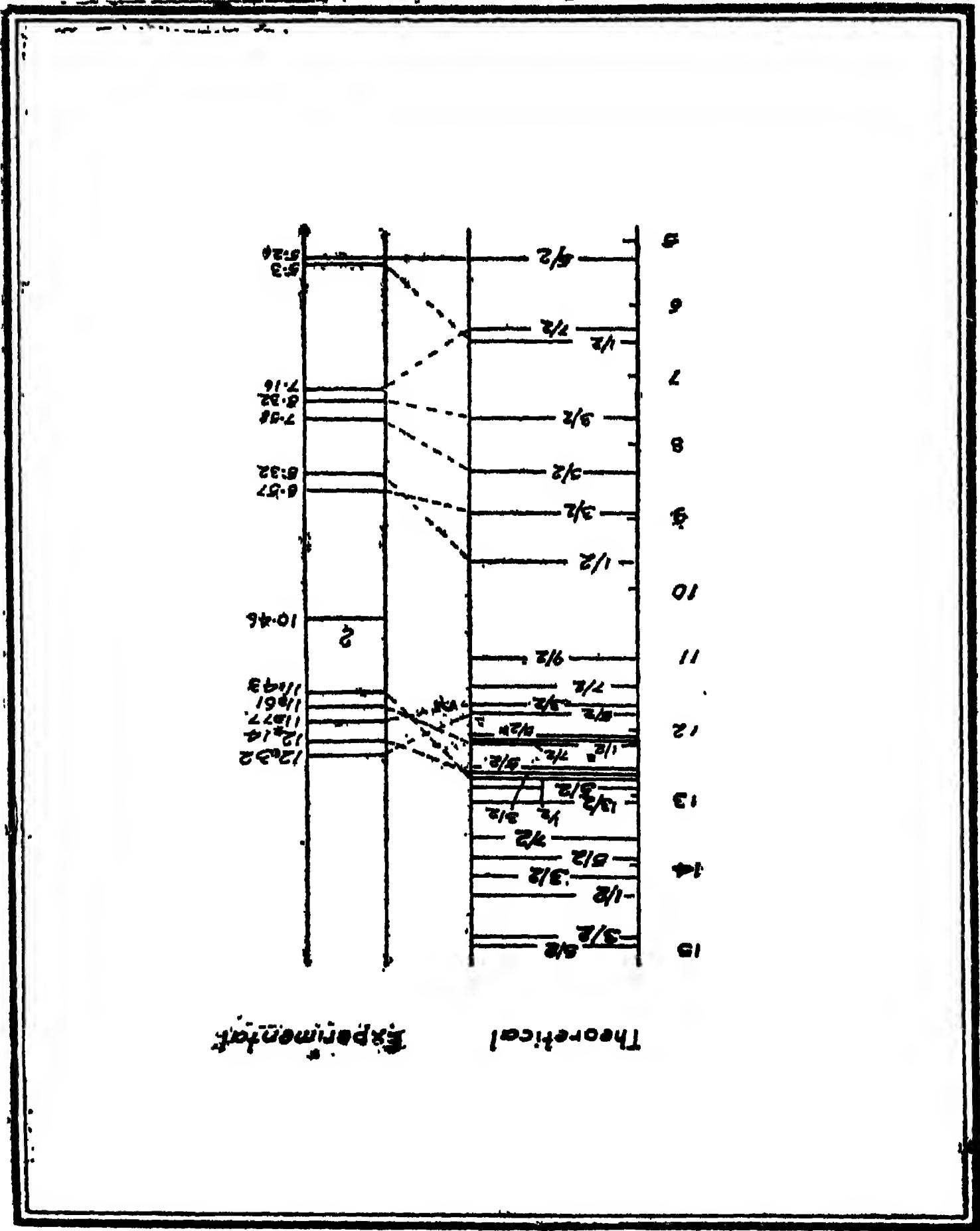


Figure 2.6

others including the ground state, being quite complicated, are different from the simple one particle shell model representation.

TABLE 2.1

L-S coupled states formed from a p-shell

K	L	(2T+1, 2S+1)	[λ]
0	0	(11)	[0]
1	1	(22)	[1]
2	0, 2	(13) (31)	[2]
	1	(11) (33)	[11]
3	1, 3	(22)	[3]
	1, 2	(22) (24) (42)	[21]
	0	(22) (44)	[111]
4	0, 2, 4	(11)	[4]
	1, 2, 3	(13) (31) (33)	[31]
	0, 2	(11) (15) (51) (33)	[22]
	1	(13) (31) (33) (35) (53)	[211]
5	1, 2, 3, 4	(22)	[41]
	1, 2, 3	(22) (24) (42)	[32]
	0, 2	(22) (24) (42) (44)	[311]
	$\frac{1}{2}$	(22) (24) (42) (26) (62) (44)	[221]
6	0, 2, 3, 4	(13) (31)	[42]
	1, 3	(11) (33)	[411]
	1, 3	(11) (33)	[33]
	1, 2	(13) (31) (33) ² (15) (51) (35) (53)	[321]
	0	(13) (31) (35) (53) (17) (71)	[222]

TABLE 2.2

j-j coupled states for $P_{3/2}$ and $P_{1/2}$ shells

K	J	$2T + 1$	(s, t)
$P_{3/2}$ shell			
0	0	1	(0,0)
1	3/2	2	(1,1/2)
2	1,3	1	(2,0)
	0	3	(0,0)
	2	3	(2,1)
3	3/2	2	(1,1/2)
	1/2, 5/2, 7/2	2	(3,1/2)
	3/2	4	(1,1/2)
4	0	1	(0,0)
	2	1	(2,1)
	2,4	1	(4,0)
	1,3	3	(2,0)
	2	3	(2,1)
	0	5	(0,0)
$P_{1/2}$ shell			
0	0	1	(0,0)
1	1/2	2	(1,1/2)
2	1	1	(2,0)
	0	3	(0,0)

TABLE 2.3

J	F_0	F_2	F_4
3/2		$\sqrt{5/7}$	$-\sqrt{5/7}$
5/2	$1/3 \sqrt{2}$	$-1/3 \sqrt{5/2}$	$-\sqrt{1/2}$
9/2		$\sqrt{3/14}$	$\sqrt{11/14}$

Independent Particle Model

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Part Three

Collective Modes of Nuclear Motion and the Unified Model

3.1 Introduction

The individual and independent particle models already dealt with in the earlier parts (I and II), consider the motion of individual nucleons in an isotropic average nuclear field produced by other nucleons in the nucleus. These are in many respects akin to the model of atomic structure which accounts for the variation of specific properties of the atoms of the individual elements. The model of atomic structure, however, differs from that of the nucleus in that, in the former, the atomic field is governed by the electrical attraction of the heavy nucleus to the extranuclear electrons of the atom. It is well known that central nucleus in the atom is of the order of 1845 times as heavy as an electron. Hence the atomic field can be treated as static. The attractive potential in the nuclear case is produced by the nucleons many of which move in orbits deviating markedly from spherical symmetry. In many nuclei a spherical overall distribution in nuclear matter may be given rise to by the existing combinations of nuclear orbits whereas in several others, the overall distribution, however, may be expected to deviate considerably from spherical symmetry. In this case the average field experienced by an individual nucleon will not be correctly given by an isotropic average nuclear potential. Moreover, the cooperative motions of many nucleons may result in the collective oscillations of the nucleus as a whole about some equilibrium shape. These collective oscillations may be expected to play an important role in the low

energy spectroscopy. A realization of the importance of the collective motion in the theory of the nucleus led to the development of the strong interaction or the liquid drop model^(1,2) which was found useful to interpret successfully many features of fission reactions.

The single particle or shell model and the strong interaction or the liquid drop model have distinct approaches to explain nuclear structure both covering the region of certain types of nuclear phenomena quite well of course with distinct limits of areas of their applicability. It was recognised that there was a necessity for the development of a model which involves the retention of the distinct features of both these approaches. With this object in view, a model was then evolved which describes the nucleus as shell structure with an addition of a static isotropic nuclear field. In this field, the motion of nucleons was substituted by a nuclear potential which allows for variation in the nuclear field resulting from the oscillations in the shape and changes spacial orientation of the nucleus. The particle motion and the nuclear oscillations in turn are mutually affecting each other. It is required to compute the interaction of these two intrinsically different types of motion and to determine their effects on the nuclear characteristics such as spin, parity, quadrupole and magnetic moments etc. This nuclear model referred as unified nuclear model which when employed exclusively for the description of spheroidal nuclei, exhibiting characteristic rotational spectra is termed as "rotational model" and when only collective motions involving the nucleus as a whole are taken into account, it is termed as collective model.

The importance of the interaction of the individual particle and collective motion phases of the nucleus were understood. As a result of a study of the quadrupole moments, it was found that the particle structure was capable of deforming nuclear surface⁽³⁾. Then theoretical investigations of a unified model of the nucleus were made⁽⁴⁾ and those of the interactions of intrinsic and collective motions of the nucleus were carried out^(5,6). Further developments were made in this direction⁽⁷⁾ and remarkable success of the unified mode in predicting the nuclear characteristics could be achieved.

The shell model as a measure of approximation ignores those nucleonic interactions which are not comprised in the average field. The liquid drop model for the nucleus implied that the nucleus was of amorphous structure. If it were so, the nucleus would be expected to have its lowest energy for a spherical shape. The individual nucleons, however, have properties characteristic to the shell model thereby implying that the nuclear shape systematically tends to distortion, the basic mechanism for which being in the effect of the capability of nuclear structure deforming nuclear surface. Hence a single nucleon moving within a nucleus exerts a centrifugal pressure against the walls of the nucleus in its orbital plane tending to produce an oblate deformation in the nuclear surface. In a nucleus with a closed shell configuration, the deforming effects of many nucleons cancel out inasmuch as the orbitals are equally oriented in all directions. The nucleus tends to adjust its surface to coincide with the density distribution of the particles which are not in filled shells. If there be no opposing force, the centrifugal pressure results in a nucleus with a space distribution equivalent to the nucleons in the unfilled shells. The effects of polarizing the closed shell core strongly preferring spherical symmetry and the pairing forces of the extra nucleons operate in the opposite direction. These effects, however, overbalance the distorting effect of the last odd nucleon and the shape of nuclear equilibrium remains spherical in the case of a nucleus with only a few nucleons beyond the closed shell. Then a decrease in the energy involved in collective vibrations about the spheroidal equilibrium shape is evident since the nucleus becomes less resistant. An addition of many particles outside the closed shell induces instability of the spherical shape resulting in the assumption of the nucleus with a spheroidal equilibrium shape. In this case, the collective motions of the nucleus will be rotational and vibrational.

These mentioned effects may be computed by replacing the symmetrical binding potential of the simple shell model with an anisotropic binding potential. Some single particle wave functions for a specific choice of an anisotropic potential has been deduced⁽⁸⁾. There remain yet the pairing forces between the nucleons outside the closed shells tending to couple two

equivalent nucleons to a state of zero angular momentum. Thus the tendency of the individual nucleons to deform the nuclear shape counteracted. The foregone discussion of the topic has as its basis the treatment of the nucleus as a core plus extracore nucleons.

This situation can be clarified by considering that the energy is separated into terms depending as well as those not depending on the collective variables, the other variables being called intrinsic. The collective degrees of freedom are important for low energy properties with the necessary preservation of the nuclear volume. The nuclear surface, then may be expressed in spherical harmonics as

$$R(\theta\phi) = R_0 \left[1 + \sum_{\lambda\mu} \mathcal{L}_{\lambda\mu} Y_{\lambda}^{\mu}(\theta\phi) \right] \quad \dots \quad (3.1)$$

R_0 being the equilibrium radius and Y_{λ}^{μ} the normalized spherical harmonic of order $\lambda\mu$. If the constants $\mathcal{L}_{\lambda}$ be assumed to be small and the frequencies of single particle excitations to be much greater than those involved in the collective motions, the total wave function can be separated into a part describing the particle motion and a part depicting the collective motion. From the above mentioned considerations, the Hamiltonian may be expressed approximately as

$$H_{\text{Collective}} = V_n(\mathcal{L}) + T_n(\dot{\mathcal{L}}) \quad \dots \quad (3.2)$$

where $V_n(\mathcal{L})$ represents the potential energy of the nucleus as a function of the shape defined by the coefficients $\mathcal{L}$, the subscript n refers to the group of quantum numbers specifying the motions of all the particles in a nucleus with shape defined by the $\mathcal{L}_n$ and the second term $T_n(\dot{\mathcal{L}})$ is the kinetic energy involved in small oscillations of the nuclear shape, $(\dot{\mathcal{L}})$ being the time derivative of $\mathcal{L}$.

The predominant term is quadratic in the $\mathcal{L}$ and the normal modes of operation are of oscillator type. The potential energy function $V_n(\mathcal{L})$ is an important basic factor of the model and its variation with nuclear shape is determined by the number of particles outside the closed shell the specific orbitals which they

fill as well as the residual nucleon-nucleon interaction of these nucleons. The potential energy function $V_n(\mathcal{L})$ varies and it increases for nuclei away from the closed shell configuration. In the case of a nucleus at a closed shell the interparticle forces of the core nucleons result in a strong preference for a spherical symmetry, the shape changes are firmly resisted and the potential energy rises steeply as a function of small deformations with addition of a few additional particles the nucleus is tended to be polarized by these particles thereby causing deformations from the spherical shape energetically more favourable.

As already mentioned the resistance of the closed shell core to polarization and the pairing forces between the extra-core nucleons, however, restrain the nucleus to a spherical ground state, the main effect of the polarizing tendency being the reduction of the rigidity toward the shape changes.

With a further addition of more particles beyond the closed shell configuration, the coherent effects of many particles result in a stabilized deformation of the nucleus in which exists a minimum potential energy for a non-spherical shape. The loose particle motion in unfilled shells is strongly coupled to the nuclear surface. A non-spherical stabilized shape can be achieved if the changes in shape associated with zero-point vibrations are small compared to the equilibrium deformation. In the case of nuclei of non-spherical type, the collective vibrations are inclusive of both vibrational oscillations and changes in orientation bringing a sort of change in shape which amounts to rotational excitation. Heavy nuclei with mass numbers greater than 225 are non-spherical in ground state. There is a substantial gain in binding energy due to deformation from the spherical shape, being of the order of 20 MeV in most strongly deformed nuclei of the rare earth region of elements⁽⁹⁾.

From a survey of the introductory remarks already given on the collective modes of motion and the unified model it can be seen that the former as a measure of simplicity, separates the nucleus into a core and extracore nucleus, the former being viewed as a deformable drops of nuclear "liquid" in interaction with a few extra nucleons in an unfilled shell. The latter one

is an extended form of shell model, the shell model potential being assumed non-spherical and the energies in non-spherical potential are calculated. The distortion yielding minimum potential is considered as the actual distortion. A study of nuclear model involves the necessity of understanding the same in terms of the essentially fundamental reactions by replacing the long range correlations by a potential assumed permanently distorted.

3.2 Classical Theory of Collective Modes of Motion

Consider a nucleus the modes of motion of which are assumed as those of an incompressible liquid, the type of its rotation being general rather than irrotational fluid flow in which each elementary volume is not rotating about an axis moving with its own center so that the orientations of its sides in space are unaltered thereby resulting in vibrational excitation modes. In such a case, let the angular moments be λ, μ and the corresponding vibrational co-ordinates be $\mathcal{L}_{\lambda\mu}$, the latter being complicated functions of nucleonic co-ordinates, moment and spin variables. Then the surface of the model of general shape of nucleus may in general be written as

$$R = R_0 \left[1 + \sum_{\lambda=0}^{\infty} \sum_{\mu=-\lambda}^{\lambda} \mathcal{L}_{\lambda\mu} Y_{\mu}^{\lambda}(\theta, \phi) \right] \dots \quad (3.3)$$

where R_0 is the equilibrium radius, θ and ϕ indicate polar angles with respect to arbitrary axis and $\mathcal{L}$ defines the nuclear surface. The collective motions may be expressed by letting $\mathcal{L}_{\lambda\mu}$ vary in time. The vibrational excitation modes may be characterised by the angular momentum quantum numbers λ, μ and the corresponding vibrational co-ordinates $\mathcal{L}_{\lambda\mu}$, the latter being complicated functions of nucleonic co-ordinates momenta and spin variables. In the case of oscillations with sufficiently small amplitudes the motion is harmonic and may be described by the Hamiltonian of the form :

$$H_{\text{vib}} = \sum_{\lambda\mu} \left[\frac{1}{2} \beta_{\lambda} |\dot{\mathcal{L}}_{\lambda\mu}|^2 + \frac{1}{2} C_{\lambda} |\mathcal{L}_{\lambda\mu}|^2 \right] \dots \quad (3.4)$$

Each mode λ , as mentioned, is characterised by mass and restoring parameters β_λ and C_λ , the frequency of the mode being

$$\omega_\lambda = \left(\frac{C_\lambda}{\beta_\lambda} \right)^{\frac{1}{2}} \quad \dots (3.5)$$

The kinetic energy T in quadratic approximation may be written as

$$T = \frac{1}{2} \sum_{\lambda, \mu} \beta_\lambda | \dot{\mathcal{L}}_{\lambda\mu} |^2 \quad \dots (3.6)$$

In the case of irrotational flow of constant density, the classical considerations show that

$$\beta_\lambda = \frac{\rho R_0^5}{\lambda} \quad \dots (3.7)$$

ρ being density. The potential energy V can be expressed as

$$V = \frac{1}{2} \sum_{\lambda, \mu} C_\lambda | \mathcal{L}_{\lambda\mu} |^2 \quad \dots (3.8)$$

In the expressions (3.4) and (3.6), β_λ and C_λ respectively are the mass and restoring force characteristics of each mode of λ . The restoring characteristic $C_\lambda^{(1)}$ for classical liquid under surface tension S may be written as

$$C_\lambda^{(1)} = S R_0^2 (\lambda - 1) (\lambda + 2) \quad \dots (3.9)$$

With the value of $C_\lambda^{(2)}$ given as

$$C_\lambda^{(2)} = \frac{3}{2\pi} \frac{z^2 e^2}{R_0} \frac{\lambda - 1}{2\lambda + 1} \quad \dots (3.10)$$

These estimates of C_λ can be quite useful provided the approximate values for R_0 and S obtained from semiempirical mass formula are employed. The restoring parameters will have a systematic A, Z dependence and discrepancies arising out of Nuclear Shell Structure can be expected. The value of β_λ derived from the expression (3.7) will not be better than an order of magnitude estimate measure inasmuch as the correct value sensitively depends on the nature of flow and the supposition

that the nuclear flow is irrotational does not seem to have any valid reason.

Collective motions are generally associated with strong electric multipole moments $M(E\lambda, \mu)$ which could be expanded in terms of collective co-ordinates. If only the leading term of the expansion is maintained, the $M(E\lambda, \mu)$ become proportional to the $\mathcal{L}_{\lambda\mu}$, the constant of proportionality depending on the normalization chosen for $\mathcal{L}_{\lambda\mu}$. The normalization can be fixed with reference to the surface oscillations of the liquid drop. The coordinates can then be defined by

$$R(\theta, \phi) = R_0 [1 + \sum_{\lambda\mu} \alpha_{\lambda\mu} Y_{\lambda\mu}^*(\theta, \phi) + O(\alpha^2)] \dots (3.11)$$

Where R_0 is equilibrium radius. The corresponding electric multipole moment M , in the case of a uniformly charged drop can be expressed as

$$M(E\lambda, \mu) = \frac{3}{4\pi} Z_0 R_0^\lambda \mathcal{L}_{\lambda\mu} \dots (3.12)$$

and in the general case of normalization, the transition probability between the first vibrational state and the ground state can be obtained as

$$B(E\lambda; n_\lambda = 1 \rightarrow n_\lambda = 0) = \left(\frac{3}{4\pi} Z_0 R_0^\lambda \right)^2 \frac{\hbar}{2 (B_\lambda C_\lambda)^{\frac{1}{2}}} \dots (3.13)$$

The vibrational excitations given by (3.4) and (3.12) are characterized by the selection rule given by

$$\Delta n_\lambda = \pm 1 \dots (3.14)$$

and by the intensity rule

$$\begin{aligned} \sum_f B(E\lambda; n_\lambda, I_i \rightarrow n_\lambda - 1, I_f) \\ = n_\lambda B(E\lambda; n_\lambda = 1 \rightarrow n_\lambda = 0) \end{aligned} \dots (3.15)$$

the sum being carried over all states with $n_\lambda - 1$ phonons, a certain I_f value, resulting in the occurrence more than once. The $B(E2; 2' + \rightarrow 2^+) / B(E2; 2 + \rightarrow 0 +) = 2$ is obtained for the second $2 +$ states in the quadrupole bands.

In the approximations the magnetic dipole operator is proportional to the angular momentum, the spin alignment being assumed to exist with this collective mode. Hence, $\mu = g_\lambda I$, the collective g-factor being the same for all branch members of a vibrational mode. Moreover, M1 transitions vanish to the extent of the same approximation.

An experimental evidence on the 2^+ states in quadrupole vibrational bands confirm a vibrational interpretation of the states involved. A remarkable interesting feature of the evidence is that the cross-over transition from the second 2^+ to the ground state is found to be strongly forbidden in conformity with the selection rule (3.14). Also the M1 components of the $2^+ \rightarrow 2^+$ gamma rays are, as expected, found to be weak. The ratios of E2 transition probabilities do not confirm the predictions of the harmonic approximation in which a triplet of quadrupole vibrational states with $I = 0^+, 2^+$ and 4^+ at an energy approximately twice that of the first 2^+ state. Such triplet spectra have been found in a number of cases among the spectra which could be investigated. The results of carefully studied spectra indicate that in some cases one or two of the triplet are missing in the concerned energy region.

The observed deviation in transition probabilities and energies from the expectations of the single harmonic oscillator description may be accounted by considering the effects of anharmonic terms^(10, 11) in the vibrational Hamiltonian. The weakness of $\Delta n_1 = 2$ transition may indicate the existence of rooms for anharmonic terms with odd powers of $\mathcal{L}_{\lambda\mu}$ and $\mathcal{L}_{\lambda\mu}$. An analysis on these lines, however, is beset with difficulties inasmuch as it involves the necessity of an inclusion of a large number of anharmonic terms in (3.4) without yielding satisfactory improvements in the description.

The magnitudes of the mass parameter β_λ and the restoring parameter C_λ are expected to fluctuate in many regions due to shell effects. The general trends averaged over the shell structure can be obtained by employing the principles underlying the liquid drop model. If the liquid drop be uniquely

charged, the modes with lowest frequency without variation in density and with preservation of neutron-proton ratio throughout the nucleus ($T = 0$), the restoring force C_λ will be as expressed by

$$C_\lambda = (\lambda - 1) (\lambda + 2) R_0^2 S - \frac{3}{2\pi} \frac{\lambda - 1}{2\lambda + 1} \frac{Z^2 e^2}{R_0}$$

with $4\pi R_0^2 S \simeq 20 A^{\frac{2}{3}} \text{ MeV}$ (3.16)

Where the first term represents the surface energy while the latter the Coulomb energy. The magnitude of β_λ depends on the assumed characteristics of the mass flow. If the motion be irrotational, the mass parameter β_λ will be expressed as

$$\beta_\lambda = \frac{1}{\lambda} \frac{3}{4\pi} A M R_0^2 \quad \dots (3.17)$$

the product AM being the mass of the nucleus. The frequency $\hbar\omega$ of the surface mode can be derived from the expressions (3.16) and (3.17) through the relation (3.5).

3.3 The Energy Weighted Sum Rule and the Strength of Vibrational Excitations

Approximate estimations of the strengths of the collective excitation are made possible by the single particle estimate of electromagnetic matrix elements even though it is model dependent. The nuclear transition strengths may in general be characterized by taking into account the consideration that the size of the radiative matrix elements is subjected to the limitation of the sum rules^(12, 13). Of the various sums, the energy of the weighted sum $\sum_f B(E\lambda) (E_f - E_i)$ is the most successful inasmuch as the corresponding theoretical unit is essentially dependent on the distribution of nuclear charge in the ground state which is known from the data of electron scattering. The theoretical units of other sum rules are strongly model dependent owing to the fact that they depend on the nucleonic correlations in the ground state.

If the effects of exchange interaction and only the sums over excitations, far below the meson threshold are considered,

the energy weighted sum rule is obtained as

$$S_{\lambda} \equiv \sum_f B(E_{\lambda}; i \rightarrow f) (E_f - E_i) \\ = \frac{\lambda(2\lambda+1)^2}{4\pi} \frac{\hbar e^2}{2M} \langle r^{2\lambda-2} \rangle \quad \dots (3.18)$$

where S_{λ} includes both $T=1$ and $T=0$ terms, the latter type being of the lowest vibrational excitations. If there be no interference between $T=0$ and $T=1$ excitations, the sum rule, which corresponds to the sum rule rate for $\lambda > 1$, is obtained as

$$S_{\lambda, T=0} = \frac{\lambda(2\lambda+1)^2}{4\pi} \frac{\hbar^2}{2M} \frac{Z^2}{A} e^2 \langle r^{2\lambda-2} \rangle \quad \dots (3.19)$$

From the available evidence on the strengths of the low lying $2+$ vibrational states, only one such vibrational excitation in even spherical nuclei could be observed with a strength of the order of $\frac{1}{20} - \frac{3}{20}$ of the total $E2$ sum rate for $T=0$ oscillation.

Hence the additional $T=0$ states with appreciable oscillator strengths are likely to occur in even-even nuclei at a much higher excitation energies. At present no such direct proof for the existence of such states is available except for the enhanced $E2$ states indirectly observed in one particle systems.

The evidence on the 3-states is incomplete. Moreover the accuracy of $B(E3)$ determinations is poorer than for quadrupole states. The total oscillator strengths found in the lowest $T=0$ modes is of the order of $1/10 - 1/50$ th of the sum rule limit. The results of observations of the strength for $E3$ excitations have been found to involve larger fluctuations than those of the observations of the strengths for $E2$ excitations. However, in those cases, where the oscillator energy of $1/10$ th the sum rule limit is found to be exhausted, the $E3$ strength is likely to be shared by more low lying states of which the lowest has so far been experimentally observed.

3.4 Vibrations in Odd Mass Nuclear Systems

If interactions are viewed in the harmonic approximation interactions between either the different vibrational quanta or

between quanta and quasi-particles do not appear to exist. Then the motion of the particles in an odd mass nucleus is unaffected by the oscillatory field set-up by the vibrations. The resulting spectrum will be comprising the intrinsic excitations which amount to the quasi-particle states as well as the series of vibrational spectra associated with the intrinsic state. The coupling of the particle state of angular momentum J and the vibrational state of angular momentum R_λ gives rise to a nearly degenerate multiplet with states of angular momenta $I = R_\lambda + J, R_\lambda + J - 1 \dots |R_\lambda - J|$ amounting to a total of $2J+1$ or $2R_\lambda + 1$ states. The energy of the multiplet relative to intrinsic state is approximately equal to the corresponding phonon energy in the neighbouring even-even nuclei. The reduced transition probability for excitation of a single quantum may be written as

$$B(E\lambda; J' \rightarrow [n_\lambda = 1, J] I) \\ = \frac{2I+1}{(2\lambda+1)(2J'+1)} B(E\lambda; n_\lambda = 0 \rightarrow n_\lambda = 1) \dots \quad (3.20)$$

Then the summing over all states I gives

$$\sum_I B(E\lambda; J' \rightarrow [n_\lambda = 1, J] I) = B(E\lambda; n_\lambda = 0 \rightarrow n_\lambda = 1) \dots \quad (3.21)$$

This theoretical presentation is nearly valid in the case of a weak coupling. Some odd mass nuclear systems are found to show strong splitting while at the same time retains several characteristics of weak coupling. In these nuclear systems there are cases which show strong coupling between the motion of the particles and the oscillatory field as well as important interactions between the elementary vibrational excitations. In these odd mass nuclear systems the low-lying states are strongly mixed quasi-particle and vibrational excitations, each state containing more than one vibrational component. It may be mentioned that a systematic interpretation of the great variety of experimentally obtained spectra of the strong coupling systems of odd mass nuclei is not at present possible.

The quadrupole strength available in a coupled system may be viewed as one being shared between the individual excitations so that the approximate sum rule⁽¹⁴⁾ may be expected to hold in such situations. Hence,

$$\sum_f B(E2; I_0 \rightarrow I_f) \geq B(E2; 0 \rightarrow 2) \quad \dots(3.22)$$

where the summed quadrupole strength in the odd-mass nuclear system is compared with the quadrupole strength in the neighbouring even-even nuclear systems. Moreover, it is required to take the sum between reasonably low energy transitions of even-even nuclear system as well as to include diagonal element from the ground state contribution. So

$$B(E2; I_0 \rightarrow I_f) = \frac{5}{16\pi} e^2 Q^2 \frac{(I_0+1)(2I_0+3)}{I_0(2I_0-1)}; I_0 \neq \frac{1}{2} \quad \dots(3.23)$$

Q being the static quadrupole moment of the ground state. The importance of the expression (3.23) may be expected to be becoming more conspicuous with the increasing coupling strength.

The evidence from the Coulomb excitation experiments on nuclei through the range of periodic system indicates that the value of

$$\sum_f B(E2; I_0 \rightarrow I_f) / B(E2, 0 \rightarrow 2) \simeq 0.6 - 0.9 \quad \dots(3.23a)$$

In many of the experimentally investigated cases there may be strong quadrupole excitations that remain undetected.

3.5 Nuclear Systems Involving Stable Quadrupole Deformations

The stable deformed nuclei of odd-mass type yield not only low-lying rotational bands with regular level spacing but also give rise to large quadrupole moments as well as isotopic shifts. At low energies ≤ 1 MeV occur several bands of rotational spectra, each band based on a particular intrinsic configuration. This characteristic justified the validity of the adiabatic separation of the nuclear wave functions into the product of D-function describing the rotation of the system as a whole and an intrinsic wave function χ . This procedure was

originally introduced by Bohr and Mottleson⁽¹⁵⁾ and the assumption that the nucleus has nearly rotational symmetry is found consistent with the type of rotational spectra encountered. Then the angular momentum component k along the nuclear symmetry axis is a constant of the motion. The χ_k intrinsic state is two fold degenerate with $k = \pm |k|$. It may be noted that many properties can be derived without further specification of the intrinsic state from only the adiabaticity and the symmetries associated with the problem.

The rotational wave function can be described in terms of the rotation matrix $D^I_{MM'}(\theta_e)$ as ^(16, 17)

$$(\Psi_{IM})_S = \sum_{M'} D^I_{MM'}(\theta_e) (\Psi_{IM'})_{S'} \quad \dots(3.24)$$

where Ψ_{IM} is the angular momentum wave function expressed in the laboratory system S , the component M being on the Z axis of the system. The function $(\Psi_{IM'})_{S'}$ refers to the system S' , momentarily coincident with the nuclear coordinate system with M' being the component of I in this system. Let the latter system be denoted by 1, 2, 3 and the Eulerian angles defining the orientation of S' with respect to S be θ_e or ϕ, θ and ψ . Then the adiabatic wave function may be written as

$$\Psi^I_{Mk} = \left(\frac{2I+1}{8\pi^2} \right)^{\frac{1}{2}} \frac{1}{\sqrt{2}} (\chi_k D^I_{Mk} + R_1 R_e^{-1} \chi_k D^I_{Mk}) \quad \dots(3.25)$$

where the second term in the bracket corresponds to an added symmetrization part to account for the reflection symmetry. The operator R_1 occurring in the added term represents a rotation of nuclear matter through 180° about the 2-axis. In this way R_1 operate on χ_k and R_e^{-1} an external rotation of the whole nuclear system, matter + axis system by 180° in the other direction. It may be pointed that this operation is carried by R^{-1} operating on D^I_{Mk} . The $R_1 R_e^{-1}$ has the effect of flipping the intrinsic 3-axis, the direction of which is undetermined corresponding to the reflection symmetry. The R_1 and R_e^{-1} operators have the properties

$$R_e^{-1} D^I_{Mk} = (-)^{I+k} D^I_{M-k} \quad \dots(3.26)$$

$$R_1 \chi_k = \chi_{-k} \quad \dots(3.27)$$

This definition of χ_{-k} has been shown to be consistent with the single particle wave function by the relation by

$$\chi_{-k} = T \chi_k \quad \dots(3.28)$$

where T is the time reflection operator. However the above definition χ_{-k} requires some modification to be made. A procedure which follows the definitions of Bohr and Mottelson yields a modification given by

$$R_i \chi_0 = \gamma \chi_0 \quad (\gamma = \pm 1) \quad \dots(3.29)$$

The intrinsic wave function χ_k which is associated with definite parity π may be defined with respect to intrinsic coordinate system S' . In the case of $K = 0$ occur two different types of rotational bands, one corresponding to $\gamma = +1$ and $I = 0, 2, 4, \dots$ and the other to $\gamma = -1$ and $I = 1, 3, 5, \dots$. The ground state band of even-even nucleus is always characterized by $K = 0$, $\gamma = 1$ and consequently $I = 0, 2, 4$ resulting from the fact that the ground state band of even-even nucleus occupies the orbits in pairs causing the same antisymmetrized. The effect of R_i on such a system is given as

$$R_i \chi_0 \equiv R_i \frac{1}{\sqrt{2}} \left[\chi_{\Omega}(1) \chi_{-\Omega}(2) - \chi_{\Omega}(2) \chi_{-\Omega}(1) \right] = \chi_0 \quad \dots(3.30)$$

In the case of $K = 0$ bands of odd-odd nuclei the situation which is different will be described at a latter stage.

3.6. Rotational spectra and perturbations together with their diagonal terms

The coupling between intrinsic motion and rotation has an effect which could be studied and interpreted in terms of rotar model. Let the angular momentum of this rotar be $\hbar R$ and let this rotar represent the core of the nucleus which is coupled to the angular momentum $\hbar j$ of a single particle to form a total $\hbar I$. The odd particle moves in the field generated by the rotar. If the field be independent of the angular velocity of the rotar, the total Hamiltonian may be

expressed as

$$H = H_{\text{intr}} + \sum_x \frac{\hbar^2 R_x}{2 \mathcal{J}_x} \quad \dots(3.31)$$

If the rotor be symmetric around its 3-axis or if there be an axis of symmetry, the moment of inertia for collective rotation about this axis vanishes which amounts to saying that if $\gamma = 0$ or π , $\mathcal{J}_3 = 0$. The moments about the other axes are equal so that $\mathcal{J}_1 = \mathcal{J}_2 = \mathcal{J}$. The rotational symmetry of the rotor also leads to R_3 . No rotation around the symmetry axis can be defined quantum mechanically. Hence $R = 0$ which amounts to the situation as the rotor becomes cylindrically symmetric $\mathcal{J} \rightarrow 0$. Consequently an excitation of this component of rotation is associated with a very large energy. A substitution of $(I - j)$ for R and an incorporation of $\hbar^2 (j^2 - j_s^2) / 2 \mathcal{J}$ into H_{intr} leads to

$$H = H'_{\text{intr}} + \frac{\hbar^2}{2 \mathcal{J}} (I^2 - I_s^2) - \frac{\hbar^2}{2 \mathcal{J}} (I_+ j_- + I_- j_+) \quad \dots(3.32)$$

where $I_{\pm} = I_1 \pm i I_2$ and

$$j_{\pm} = j_1 \pm i j_2$$

The expression (3.25) can be seen to be an approximate Eigen function of the Hamiltonian of (3.32) provided that the last term can be treated small. The sum of the intrinsic energy E_k and a rotational energy term proportional to $I(I + 1)$ is its energy. The coupling scheme corresponding this situation may be illustrated as shown in Fig. 3.1. It may be noted that the total angular momentum I has the component M along the fixed axis z and the component K along the nuclear symmetry axis 3. The collective rotational angular momentum R is at right angles to the symmetry axis.

The perturbation neglected in (3.32) corresponds to the well-known Coriolis interaction energy which in its lowest order gives rise to couplings between different rotational bands with $\Delta K = \pm 1$. In the case of $K = \frac{1}{2}$, the two parts of the total wave function (3.25) may respectively be coupled with $K = \frac{1}{2}$ and $K = -\frac{1}{2}$ owing also to its possession of the diagonal elements, explaining thereby

that $K = \frac{1}{2}$ bands strikingly deviate from the simple effective $I(I+1)$ spacing so well-obeyed by other bands as can be seen from Fig. 3.2.

The coupling effects may be expressed in general terms as the sum of intrinsic energy and rotational energy which is expressed as the total energy. The rotational energy should be a small quantity inasmuch as the rotational motion is slow. The additional energy resulting from rotation may be expanded as a power series in the total angular momentum. The construction of the terms of the involved expansion is based on the conditions that the total Hamiltonian is invariant with respect to time reflection and rotations about the 3-axis and also is Hermitian giving rise to the three terms as expressed by (3.33), (3.34), (3.35) and (3.36) of which the first is given as

$$H_{\text{rot}}^{(1)} = h_0(q) (I_1^2 + I_2^2) \quad \dots(3.33)$$

$h_0(q)$ being operative on the intrinsic wave function without change of the angular momentum K and I_1 and I_2 on the D-function. There will be the occurrence of similar terms proportional to K^2 which however, are constants of motion, being the same for all members of the band could usually be left out of discussion.

Moreover, the diagonal matrix elements $h_0(q)$ can have an expression which may be considered capable of defining the moment of inertia is given as

$$\langle K | h_0 | K \rangle = \langle -K | h_0 | -K \rangle \equiv \hbar^2/2\mathcal{J} \quad \dots(3.34)$$

the other terms, terms of low order in I being

$$H_{\text{rot}}^{(2)} = h_{+1} I_- + h_{-1} I_+ \quad \dots(3.35)$$

and

$$H_{\text{rot}}^{(3)} = h_{+2} I_-^2 + h_{-2} I_+^2 \quad \dots(3.36)$$

which give rise to couplings between intrinsic and rotational motion. They possess diagonal matrix elements with respect to the wave function (3.25) only if $K = \frac{1}{2}$ and $k = 1$ respectively. If $K = 1$, the intrinsic parameters a and b in terms

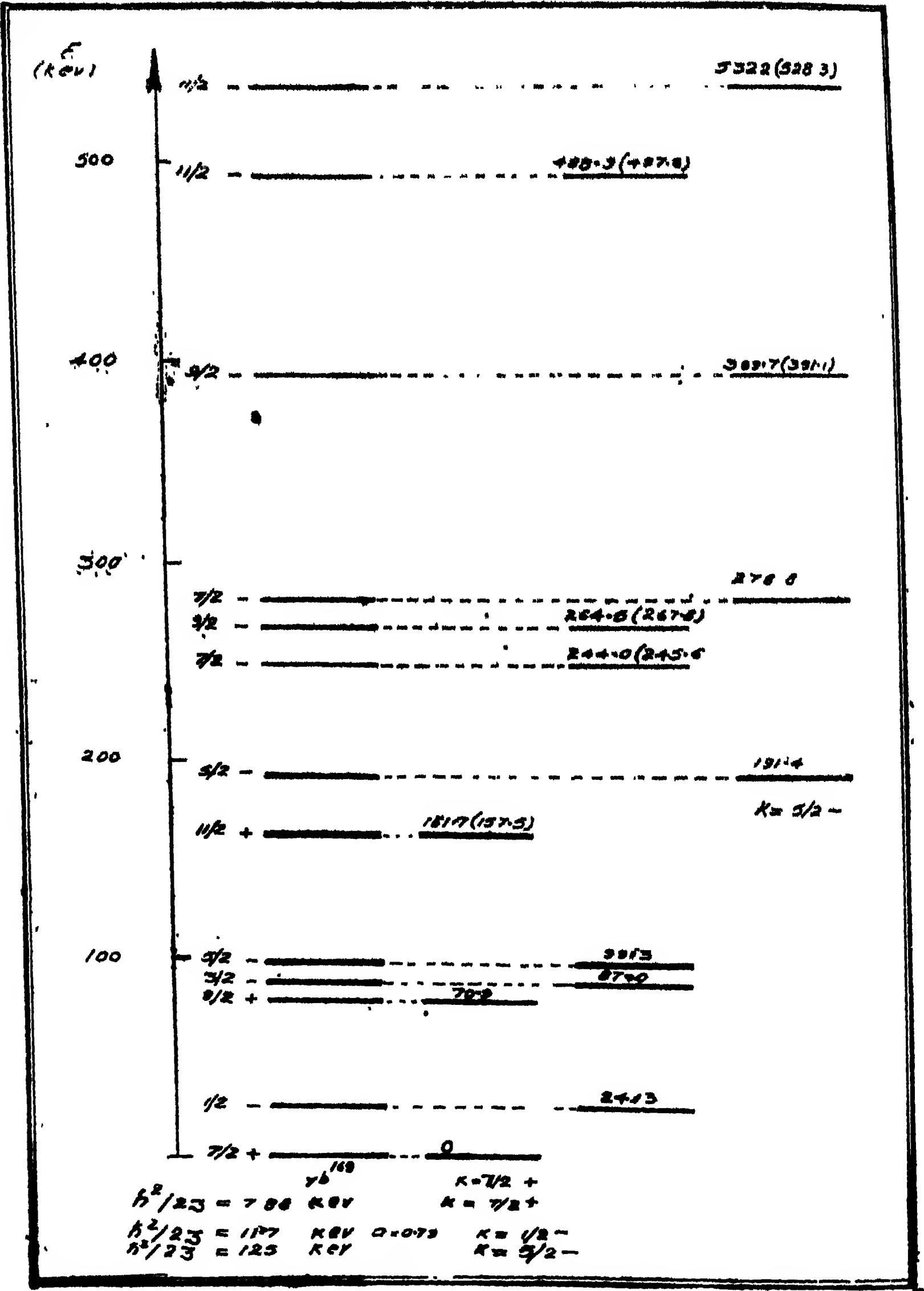


Figure 3.2

The Energy Spectrum of Yb^{169} has been studied from the Electron Capture of Lu^{169}

of the matrix elements $h_{\pm 1}$ and $h_{\pm 2}$ may be defined as

$$\begin{aligned} \langle K=\frac{1}{2} | h_{+1} | K=-\frac{1}{2} \rangle &= \langle K=-\frac{1}{2} | h_{-1} | K=\frac{1}{2} \rangle \dots (3.37) \\ &\equiv (\hbar^2/2\mathcal{J}) a \end{aligned}$$

and

$$\begin{aligned} \langle K=1 | h_{+2} | K=-1 \rangle &= \langle K=-1 | h_{-2} | K=1 \rangle \\ &\equiv (\hbar^2/2\mathcal{J}) b \dots (3.38) \end{aligned}$$

Hence, the part of the rotational energy that is diagonal with respect to the wave function (3.26) may be expressed as

$$\begin{aligned} E_{\text{rot}} = \frac{\hbar^2}{2\mathcal{J}} \left[I(I+1) + \delta_{K, 1/2} a (-)^{I+\frac{1}{2}} (I+\frac{1}{2}) \delta_{K, 1} \right. \\ \left. b (-)^{I+1} I(I+1) \right] \dots (3.39) \end{aligned}$$

The matrix elements on the basis of simple rotor model may be written as

$$\begin{aligned} h_{\pm 1} &= -(\hbar^2/2\mathcal{J}) j_{\pm} , \\ h_{\pm 2} &= 0 \end{aligned}$$

The intrinsic parameter a in the rotor model is defined as the decoupling factor given by

$$a = - \langle K=\frac{1}{2} | j_{+} | K=-\frac{1}{2} \rangle \dots (3.40)$$

forming the measure of the extent of decoupling of the intrinsic spin from the collective rotation so that an addition of $s_{1/2}$ particle to a rotor yields $a=1$. Consequently the expression (3.39) gives a spectrum with the energy levels, $I=\frac{1}{2}, (2\pm\frac{1}{2}), (4\pm\frac{1}{2}), (6\pm\frac{1}{2}) \dots$ with energy spacing same as that of an $I=0, 2, 4, 6 \dots$ band. Therefore the odd-particle does not at all seem to contribute to the rotational energy of the system thereby being completely decoupled. A comparison of the experimental and theoretical values of the decoupling factor corresponding to various configurations and the fact that $h_{\pm 2}$

is zero show that this contribution to be small. $H^{(3)}_{\text{rot}}$ may be considered to be representing some of the higher order effects $H^{(2)}_{\text{rot}}$ or Coriolis term. It is at present not possible to ascertain how much of the empirical energy correction term of the

type displayed by the last term in (3.39) is due to the diagonal effects of $H^{(3)}_{\text{rot}}$ and how much to second order effects of $H^{(2)}_{\text{rot}}$.

It may be mentioned that many rotational bands of both even-even and odd-A nuclei are found to follow the law expressed by (3.39) with remarkable accuracy. The energy ratios of ground state bands of even-even nuclei have been found to have systematic deviations which could be explained in terms of the rotational perturbation terms. The rotational bands of even-even nuclei are observed to extend upto $I=6$, 12 and 12 in the regions I, II and III respectively. The collective rotations are constructed from the available single-particle orbitals of given intrinsic spins under the restrictions of the Pauli principle. Hence only limited values of I can occur for excitation energies that are low compared with $2 \hbar \omega_{\text{osc}}$. The limit in the regions II and III or I is almost of a magnitude above the highest I so far known. The limit on I , however, is expected to be much lower on account of a possible sudden break down of the nuclear superfluidity at a critical rotational frequency corresponding to I -values of the order of or slightly higher than those so far observed⁽¹⁸⁾. The states with I -values higher than I_{crit} are associated with another intrinsic structure as well as nearly rigid moment of inertia. The E2 transition probability connecting the band members with $I > I_{\text{crit}}$ with members having $I < I_{\text{crit}}$ will sustain a reduction by a factor which could be of the order of 10–100, thereby could be expected to be reduced down to single particle value.

3.7 The Separability of the Nuclear Wave Function

In the adiabatic limit the nuclear wave function could be separated into a product of one intrinsic and one rotation factor. Such separability implies general relations of essentially geometric character for matrix elements when there is an involvement of the members of one and the same rotational band. The general procedure of the evaluation of transition matrix elements consists of transforming any multiple operator $\mathcal{M}(\lambda, \mu)$ defined in the laboratory system to the coordinate system fixed

in the nucleus as

$$\mathcal{M}(\lambda, \mu) = \left[\sum_{\nu} D^{\lambda}_{\mu\nu}(\theta_i) \mathcal{M}'(\lambda, \nu) \right] \text{sym} \quad \dots(3.41)$$

$\mathcal{M}'(\lambda, \nu)$ being the moment as observed from intrinsic coordinate system and symmetrization therein being implied is important only if $\mathcal{M}'$ contains operators that do not commute with $\mathbf{D}$ such as the collective part of the magnetic operators which is proportional to $\mathbf{I}$. The reduced matrix element independent of $\mathcal{M}$ and of the orientation of the laboratory coordinate system in terms of symmetrized nuclear wave function can be written as

$$\begin{aligned} \langle I_f K_f \| \mathcal{M}(\lambda) \| I_i K_i \rangle = & \\ (2 I_i + 1)^{\frac{1}{2}} \{ & \langle I_i K_i \lambda (K_f - K_i) | I_f K_f \rangle \\ & \langle K_f | \mathcal{M}'(\lambda, K_f - K_i) | K_i \rangle + \\ & (-1)^{I_i + K_i} \langle I_i(-K_i) \lambda K_i + K_f | I_f K_f \rangle \\ & \langle K_f | \mathcal{M}'(\lambda, K_i + K_f) | -K_i \rangle \} \end{aligned} \quad \dots(3.42)$$

The relation between the reduced transition probability and the reduced matrix element may be expressed as

$$B(\lambda; I_i K_i \rightarrow I_f K_f) = (2 I_i + 1)^{-1} \langle I_f K_f \| \mathcal{M}(\lambda) \| I_i K_i \rangle^2 \quad \dots(3.43)$$

If only one of the bands has $K=0$ but not both then, only the first term for the reduced matrix element in (3.42) occurs with a multiplication factor $\sqrt{2}$

$$\begin{aligned} \langle I_f K_f \| \mathcal{M}(\lambda) \| I_i K_i \rangle = & \sqrt{2} (2 I_i + 1)^{\frac{1}{2}} \langle I_i K_i \lambda (K_f - K_i) | I_f K_f \rangle \\ & \langle K_f | \mathcal{M}'(\lambda, K_f - K_i) | K_i \rangle \end{aligned} \quad \dots(3.44)$$

If $\lambda < |K_i - K_f|$, both the terms in (3.42) will be found to cease to exist, this cessation being known as K -forbiddenness. The degree of forbiddenness for β and γ transitions in the deformed region in each case is empirically associated with a hindrance factor of 10–100. In the case that $\lambda < K_i + K_f$, the last term occurring in (3.42) vanishes so that in the other cases that $|K_i - K_f| \leq \lambda < K_i + K_f$, the branching from an arbitrary state to any two members of one rotational band is independent of any intrinsic matrix element and depends only on a ratio of two Clebsch-Gordan coefficients⁽¹⁹⁾. The determination of K -value of excited states has been made possible by employing

the mentioned branching rules. The relation between the spectroscopic quadrupole moment Q and the intrinsic quadrupole moment Q_0 can be obtained by employing the relation given by (3.41) as

$$Q = \frac{3 K^2 - I(I+1)}{(I+1)(2I+3)} Q_0 \quad \dots(3.45)$$

The values of Q and Q_0 are experimentally well-verified. However the values of the latter are at present found to be more accurate than those of the former. The intrinsic quadrupole moment Q_0 is so related to the nuclear eccentricity parameter δ that the latter could be found by the relation given by

$$Q_0 = \frac{4}{3} Z \langle r \rangle^2 \delta \quad \dots(3.46)$$

Moreover, there is the case of charge distribution being everywhere the same between two arbitrarily closed equipotential surfaces. In such a case the eccentricity parameter δ is related to the eccentricity coordinate ε defined for non-isotropic harmonic oscillator potential by an approximate relation given by

$$\delta \simeq \varepsilon (1 + \frac{1}{2} \varepsilon) \quad \dots(3.47)$$

The nuclear eccentricity parameter ε as defined by (3.46) and (3.47) can be empirically determined from empirical Q_0 values. The distortion parameter ε versus the stable nuclei all through the rare earth region is plotted as shown in Fig. 3.3.

The magnetic dipole operator contains some part proportional to $I_{\pm}$ which represents the magnetic moment resulting from rotating nucleonic charges and containing the collective Gyromagnetic ratio g_R . Hence, it is simpler than this M1 operator. A term containing $I_{\pm}$ does not commute with the D-function and can be easily evaluated in the laboratory system. The $\mathcal{M}(M1, \mu)$ expressed in terms of a simple model of a rotor with $K=0$ coupled to an odd particle can be written as

$$\begin{aligned} \mathcal{M}(M1, \mu) = & \left(\frac{3}{4\pi} \right)^{\frac{1}{2}} \frac{e\hbar}{2Mc} \left[g_R I_{\mu} + \right. \\ & \left. + \sum_{\nu} \{ (g_I - g_R) I_{\nu} + (g_s - g_R) S_{\nu} \} D^1_{\mu\nu}(\theta_i) \right] \quad \dots(3.48) \end{aligned}$$

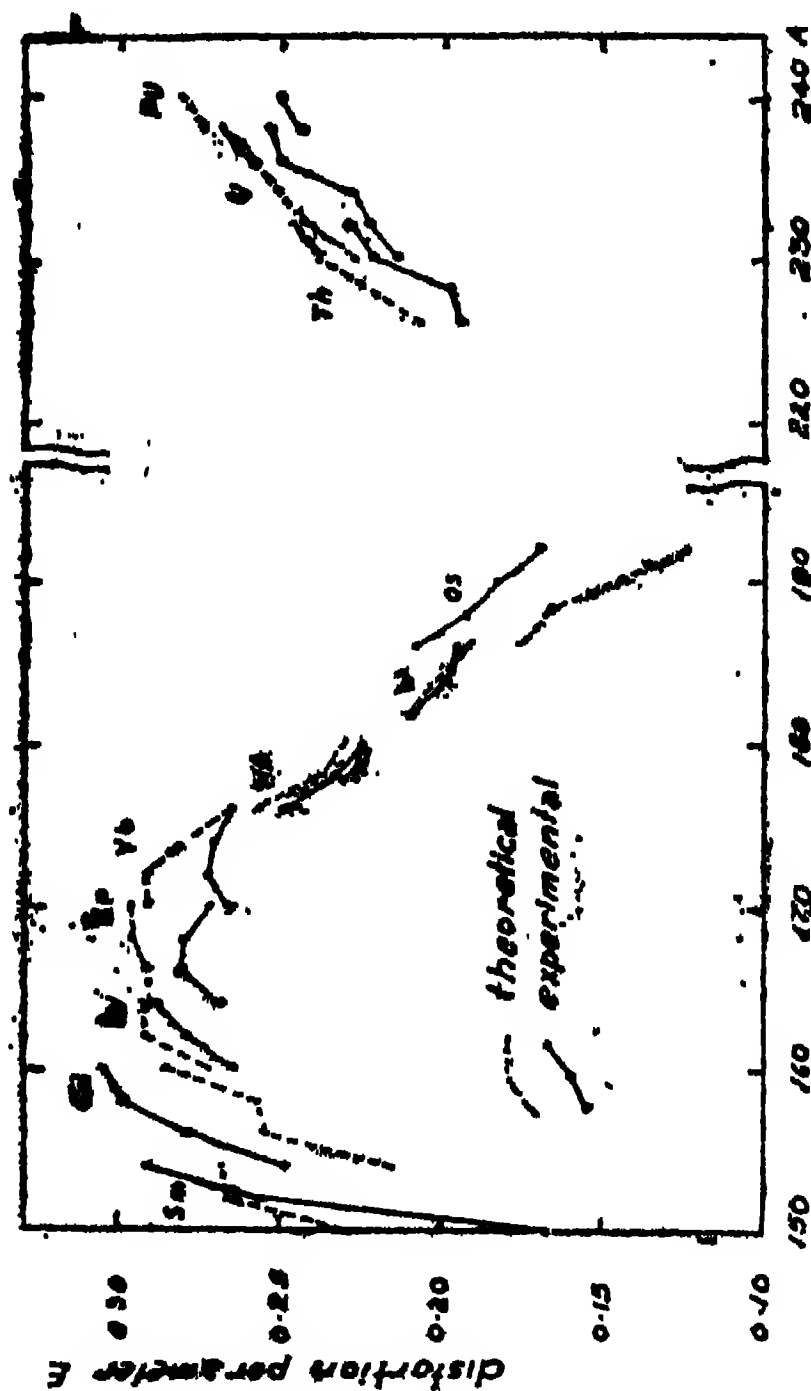


Figure 3.3
The Quadrupole Deformation Parameter ϵ from Empirical
Values from Intrinsic Quadrupole Moments Q_0

In this case the nuclear magnetic moment can be proportional to the diagonal part of the zero component of the operator. The magnetic moment and the M1 transition within an odd A rotational band can be expressed in terms of the intrinsic parameters g_K and b_0 . These parameters are entirely of single particle model in addition to the collective Gyromagnetic ratio g_R occurring in (3.48). The intrinsic parameter term b_0 , however, takes place only for $K = \frac{1}{2}$. Accordingly,

$$\mu = g_R I + (g_K - g_R) \frac{K^2}{I + 1} \left\{ 1 + \delta_{K, \frac{1}{2}} (2I + 1) (-1)^{I + \frac{1}{2}} b_0 \right\} \quad \dots(3.49)$$

and

$$B(M1; I_i K \rightarrow I_f K) = \frac{3}{4\pi} \left(\frac{e\hbar}{2Mc} \right)^2 < I_i K 10 | I_f K >^2 K^2 (g_K - g_R)^2 \left(1 + \delta_{K, \frac{1}{2}} b_0 (-1)^{I_{>} + \frac{1}{2}} \right)^2 \quad \dots(3.50)$$

where

$$g_K = K^{-1} \left\{ K_{gI} + (g_s - g_l) < K | S_z | K > \right\} \quad \dots(3.51)$$

and

$$(g_K - g_R) b_0 = (g_l - g_R) < K = \frac{1}{2} | j_+ | K = -\frac{1}{2} > + (g_s - g_l) < K = \frac{1}{2} | S_+ | K = -\frac{1}{2} > \quad \dots(3.52)$$

The g_K and b_0 are magnetic quantities which may be expressed in terms of the single particle wave function⁽²⁰⁾. $I_{>}$ occurring in (3.50) represents the larger of I_i and I_f .

The angular correlation studies involving M1 and E2 transitions show an additional quantity δ_K and the ratio of the E2 to the M1 transition amplitude which are subject to measurement. Moreover

$$\text{Sign } \delta = \text{Sign } \frac{g_K - g_R}{Q_0} (1 + b_0 (-1)^{I_{>} + \frac{1}{2}} \delta_{K, \frac{1}{2}}) \quad \dots(3.53)$$

If the sign of the quadrupole moment Q_0 be known, the measurement of μ , $B(M1)$ and δ ascertains the magnetic quantities g_K and g_R uniquely for $K \neq \frac{1}{2}$. In the case $K = \frac{1}{2}$, the value of $B(M1)$ of an additional transition within the band or the magnetic moment of a rotational state is required to determine the three magnetic quantities g_K , g_R and b_0 . If π be the parity of the odd particle state, the relation between the matrix elements of s_+ and s_3 may be expressed in terms of the single particle functions in the form given by

$$\langle \frac{1}{2} | S_+ | -\frac{1}{2} \rangle = -\pi (\frac{1}{2} + \langle \frac{1}{2} | S_3 | \frac{1}{2} \rangle) \quad \dots(3.54)$$

so that the relation between b_0 , α , g_R and g_K are given rise to which are independent of the detailed nuclear wave functions as explained by Bohr and Mottelson as well as by Nilsson. Consequently (3.52) may then be rewritten as

$$(g_K - g_R) b_0 = - (g_l - g_R) \alpha - \frac{1}{2} \pi [g_s - 2g_l + g_K] \quad \dots(3.55)$$

Employing the empirical values of α , g_R and g_K along with the effective values of g_s , the expression (3.53) may be utilized for testing the single particle description.

3.8 The Terms of Non-adiabatic Coupling

The empirical data so far obtained, if analyzed in terms of laws theoretically derived, unequivocally reveal a remarkable agreement that gives an evidence on the validity of the simple axially symmetric coupling scheme enabling there by the establishment of validity of the range of axially symmetric coupling scheme. The rotation spectra are found to confirm the expression for odd-A and odd-odd nuclei as well as for even-even nuclei the expression given by (3.39) the number of illustrative cases for odd-A spectra being given in Fig. 3.2 and Fig. 3.4 for deformed regions respectively. In the case of odd-A and odd-odd nuclei of deformed regions II and III, moments of inertia are in general larger, varying much less smoothly with A than in the case of deformed regions of even-even nuclei. There are a few cases in which occurs difference by a factor of approximately two indicating strong coupling effects resulting from the coupling terms of (3.35) and (3.36) being neglected. Nevertheless the branching rules for transitions of single particle strength, mainly $M1$ or of collective

character E2 are obeyed quite well whereas in the case of highly hindered transitions such as for, low energy E1, the rules, as expected, are often strongly violated.

Now it may be shown that a renormalization of some of the basic collective parameters as $\mathcal{J}$, g_R , etc, can be shown to account some of the main effects of the rotational perturbation terms⁽²¹⁾.

3.9 Treatment to Lower Order of Coupling Terms with $K=1$

There are terms H_{rot} , and M^s_{rot} , giving rise to coupling effects of which only the diagonal contributions in section 6 have been considered. There in H_{rot} has been proved to be very probably the larger term so that a further study restricted to this case is undertaken as follows beginning with a definition of

$$H^2_{rot} = h_{+1} I_- + h_{-1} I_+$$

which in the rotor model being

$$h_{+1} = - \frac{\hbar}{2\mathcal{J}} J_+$$

This term may be taken into account to first order in order to receive the $K = 0$ ground state band accepting the component of admixture of the form,

$$| I, K = 0 \rangle = | I, K = 0 \rangle_0 - \sum_i \sqrt{\mathcal{J} I (I + 1)} \epsilon_i | I, K = 1, i \rangle_0 \quad \dots(3.56)$$

In this case then states with K different by unity but with the same spin and parity are admixed, the reduced admixture amplitudes E_i , being given as

$$\epsilon_i = \frac{1}{\epsilon_1 - \epsilon_0} \langle \chi_i | h_{+1} | \chi_0 \rangle \quad \dots(3.57)$$

ϵ_1 being the total energy of the intermediate state and ϵ_0 that of the ground state whereas the corresponding second order corrections to the total energies being given as

$$\delta E = - \sum_i 2 (\epsilon_1 - \epsilon_0) | \epsilon_i |^2 I (I + 1) \quad \dots(3.58)$$

Then expressing this correction as a change of the moment of inertia $\delta \mathcal{J}$ in the simple rotor case may be written as

$$\delta \mathcal{J} = \hbar^2 \sum_i \frac{|\langle \chi_i^i | J_+ | \chi_0 \rangle|^2}{\epsilon_1 - \epsilon_0} \quad \dots(3.59)$$

Similarly the change in g_R can be written as

$$\delta g_R = \frac{\hbar^2}{2\mathcal{J}} \sum_i \frac{\langle \chi_i | \mu_- | \chi_1^i \rangle \langle \chi_1^i | J_+ | \chi_0 \rangle}{\epsilon_1 - \epsilon_0} \quad \dots(3.60)$$

It can actually be shown that all of the moment of inertia and magnetization may be regarded as sums of contributions from (3.56) and (3.57).

A renormalization of $\mathcal{J}$ for the band energies of g_K and g_R for M1 moments as well as of Q_0 for the E_2 moments, the first order effects may be included except for $K = 1$ or $\frac{1}{2}$, the situation in which case may to some extent become complicated thus giving scope to the necessity for disregarding the terms of the order of Q_{sp}/Q_0 .

If the transitions between two different bands with K -values differing by unity, then even the branching rules apply to first order as far as $E2$ transitions are concerned although $\mathcal{M}(E2)$ is renormalized. The coupling terms discussed, however, destroy the branch rules for M1 transitions. The situation on the other hand again is simplified for M1 transitions between two bands with $\Delta K = 2$. The coupling term relaxes the K -selection rule enabling the single branching rules to be applied to the extent that H_{rot}^2 is taken as the only important coupling term expressed in terms of $B(M1)$ as

$$\begin{aligned} B(M1; I_i K \rightarrow I_f K \pm 2) &= \\ &= \text{Const.} \times (I_i \mp K) (I_i \pm K + 1) \\ &\quad \langle I_i (K \pm 1) \frac{1}{2} (\pm 1) | I_f K \pm 2 \rangle^2 \end{aligned} \quad \dots(3.61)$$

Let H_{rot}^2 term in the case of $K = 0$ be taken to higher order giving correction terms to the usual rotational energy. With the inclusion of these effects, the E for $K = 0$ may be written as

$$E_0 = A I(I+1) + B I^2(I+1)^2 + C I^3(I+1)^3 + D I^4(I+1)^4 + \dots(3.62)$$

Whereas for $K = \frac{1}{2}, \frac{3}{2}, 1$ and 2 , terms containing 1 part with $\frac{1}{2} = K$ and $I = -K = 0$ are obtained in addition to the B , C , and D terms so that

$$\Delta E_{\frac{1}{2}} = (-)^{I+1/2} (I + \frac{1}{2}) [A_1 + B_1 I(I+1) + \dots] \quad \dots(3.62a),$$

$$\Delta E_{\frac{3}{2}} = (-)^{I+3/2} (I - \frac{1}{2}) (I + \frac{1}{2}) (I + \frac{3}{2}) [B_3 + C_3 I(I+1) + \dots] \quad \dots(3.62b),$$

$$\Delta E_1 = (-)^{I+1} (I+1) [A_2 + B_2 I(I+1) + \dots] \quad \dots(3.62c),$$

$$\Delta E_2 = (-)^I (I-1) I (I+1) (I+2) [B_4 + C_4 I(I+1) + \dots] \quad \dots(3.62d),$$

The term H^2_{rot} represents Coriolis coupling. If only Coriolis coupling alone is considered, the terms occurring in equations (3.62a) to (3.62d) will have the following magnitudes. Thus A_1 will turn out to be the usual decoupling term A . B_1 and B_3 will be of third order in H^2_{rot} and B_2 and B_4 will be of fourth order in H^2_{rot} . The coupling term for $K = 1$, i.e., H^2_{rot} will contain a diagonal term of the same I -dependence as A_2 -term. The rotational energy due to H^3_{rot} is

$$\propto K, I b(-)^{I+1} I(I+1).$$

In the case of $K = \frac{1}{2}$ and $K = \frac{3}{2}$ energy levels, if coupling terms of the type H^1 and H^2_{rot} are taken into account, a one-step lower order perturbation effect alone need be considered as against the second order effect of H^2_{rot} taken into account in $K=0$ energy level. So far as typical intrinsic energy differences are considered, H^3_{rot} is of the same order as $[H^2_{\text{rot}}]^2$. Further non-diagonal terms of H^1_{rot} will be still smaller.

A comparison of theoretically computed values⁽²²⁾ taking into account only H^2_{rot} with the empirical values of B appearing in rotational energy expansion does indicate some quantitative agreement. A significant comparison can be made by the improvement of the calculations with the inclusion of pairing. The B_3 term has been identified⁽²³⁾ in the spectrum of Tb^{159} .

An analysis of the measured energy values of the band components requiring more states with the necessity of the inclusion of more parameters, this inclusion, however, being

restricted to the fact that rotational bands are expected to terminate or drastically change character at higher angular momenta. Now the generalized intensity rule can further be generalised to hold for other multipoles in the case of an arbitrary order of K-forbiddenness $\Delta K - \lambda \geq 100$ as

$$\begin{aligned}
 B(\lambda; I_1 K \rightarrow I_2 K + \Delta K) &= \\
 &= \text{Const.} \times \langle I_1 \lambda K + \Delta K - \lambda \lambda | I_2 K + \Delta K \rangle^2 \times \\
 &\quad \times \frac{(I_1 - K)! (I_1 + K + \Delta K - \lambda)!}{(I_1 + K)! (I_1 - K - \Delta K + \lambda)!} \quad \dots(3.63)
 \end{aligned}$$

The experimental verification of the approximate validity of the equation (3.63) although sufficiently adequate for its justification, the M1 branchings from the $K = 7^+$ band to the $K = \frac{1}{2}^+$ band in Tm^{169} have been found as to warrant its validity to within 50%. In this context it may be mentioned that on the basis of the fact that the leading order intrinsic element is hindered depends on the convergence of the Coriolis expansions behind this equation.

3.10 Intrinsic Particle Levels in Strongly Deformed Nuclei

The intrinsic motion of nucleons is determined to first order by an average field effect as in the case of spherical nuclei. The observations of rotational spectra in nuclei confirm the fact that deformations have a cylindrical symmetry and hence it will suffice to consider particle states in the fields of spheroidal type.

An oscillator potential with spheroidal shape distortion with some terms to reproduce shell model levels can be introduced as

$$V = \frac{1}{2} M [\omega_{\perp}^2 (x^2 + y^2) + \omega_s^2 z^2] + C \vec{l} \cdot \vec{s} + D \vec{l}^2 \quad \dots(3.64)$$

$$\omega_s = \omega_0 (1 - \frac{2}{3} \epsilon) \quad \dots(3.65)$$

$$\omega_{\perp} = \omega_0 (1 + \frac{1}{3} \epsilon) \quad \dots(3.66)$$

ϵ being the deformation parameter, C determining the strength of spin orbit and D the strength of the square well potential

Introducing $V = \pi \hbar \omega_0 R$

where $R = \eta V - 2 \overrightarrow{l} \cdot \overrightarrow{s} - \mu l^2$, the eigen values of R gives the various energy levels in a deformed potential. Each energy level is characterized by quantum numbers N, L, Ω and still is a function of Nilsson deformation parameter η given by

$$\eta = \frac{\xi}{x} \left[1 - \frac{4}{3} \xi^2 - \frac{16}{27} \xi^3 \right]^{-\frac{1}{6}} \quad \dots (3.67)$$

The representation of $E [\hbar \omega_0(\xi)]$ versus η brings out well the intrinsic energy levels of particles in a deformed potential. Since $\hbar \omega_0(\xi)$ depends on mass number the level picture varies with mass number as can be noted from figure 3.5 to figure 3.9. The wave functions to this potential with the inclusion of the $\overrightarrow{l} \cdot \overrightarrow{s}$ and $\overrightarrow{l}^2$ terms computed from Nilsson's theory ⁽²⁴⁾ and the corresponding energy levels for the regions of nuclei where the deformed coupling scheme appears to have been satisfactorily established are represented in Figs. 3.5 to 3.9. The asymptotic wave functions are strictly valid only within the limit where the anisotropy term of the harmonic oscillator is dominant and the $\overrightarrow{l} \cdot \overrightarrow{s}$ and $\overrightarrow{l}^2$ terms are included only to the lowest order. Ignoring the $\overrightarrow{l} \cdot \overrightarrow{s}$ and $\overrightarrow{l}^2$ terms occurring in the expression (3.64), the energy may be expressed as

$$E(n_x, n_y, n_z) = (n_z + \frac{1}{2}) \hbar \omega_z + (n_{\perp} + 1) \hbar \omega_{\perp} \quad \dots (3.68)$$

where $n_{\perp} = n_x + n_y$. The angular momentum component along the nuclear symmetry axis Λ and the spin component Σ can classify the degenerate states of equal value of $n_{\perp}$, the $\Lambda + \Sigma = \Omega$, the quantity may be taking the values $\Lambda = n_{\perp}, n_{\perp} - 2, \dots, 0$ or 1 .

In the case of this study, it is more convenient to express wave functions as combinations of $[N n_z \Lambda \Sigma]$ than Nilsson's basic vectors $(N L \Lambda \Sigma)$. In these combinations $N = n_z + n_{\perp}$, the

diagonal components of the terms $\overrightarrow{l} \cdot \overrightarrow{s}$ and $\overrightarrow{l}^2$ are large but the diagonal elements are small thereby causing only minor corrections to the energy levels as well as to the wave functions.

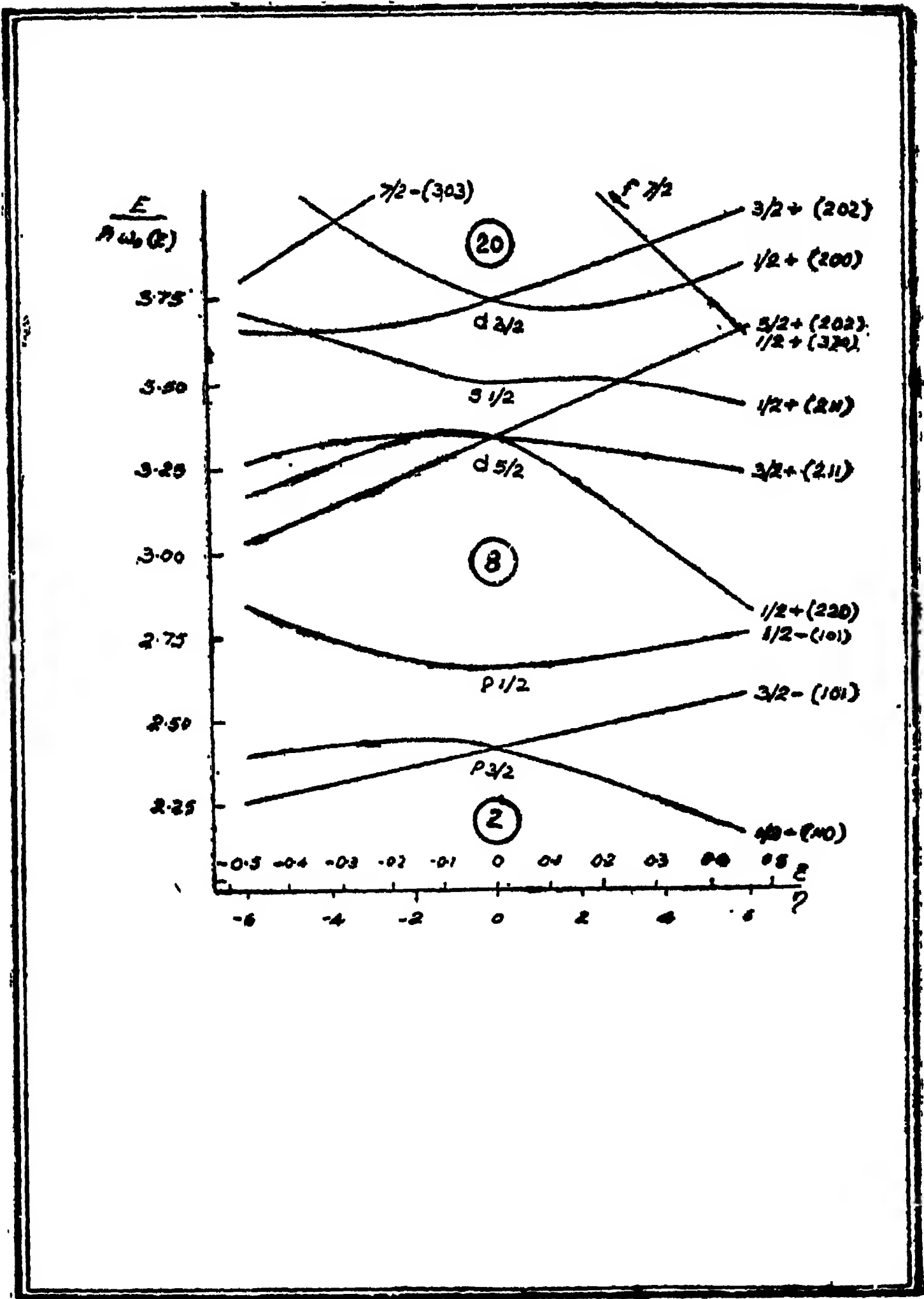


Figure 3.5

Single Particle Levels in the Region $8 < Z < 20$
and $8 < N < 20$

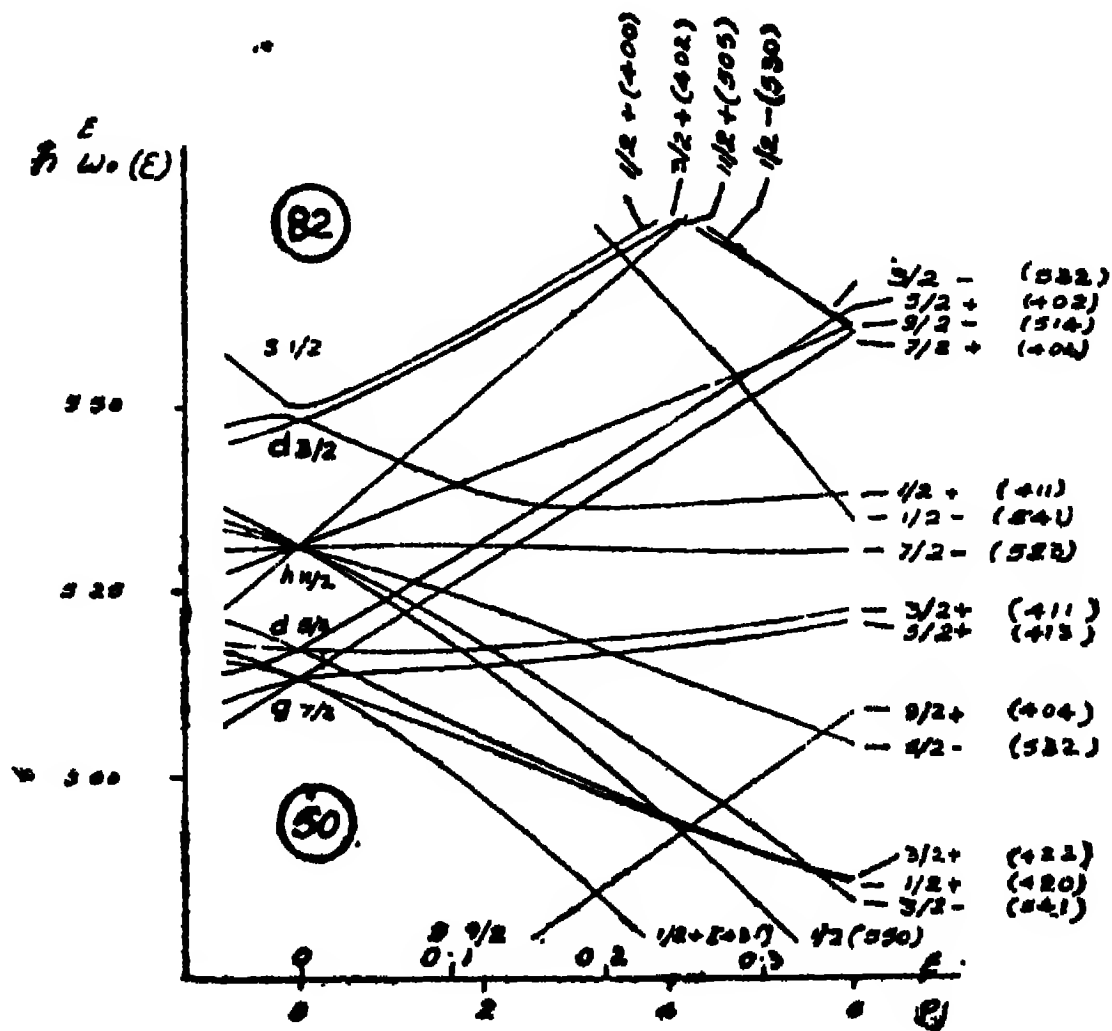
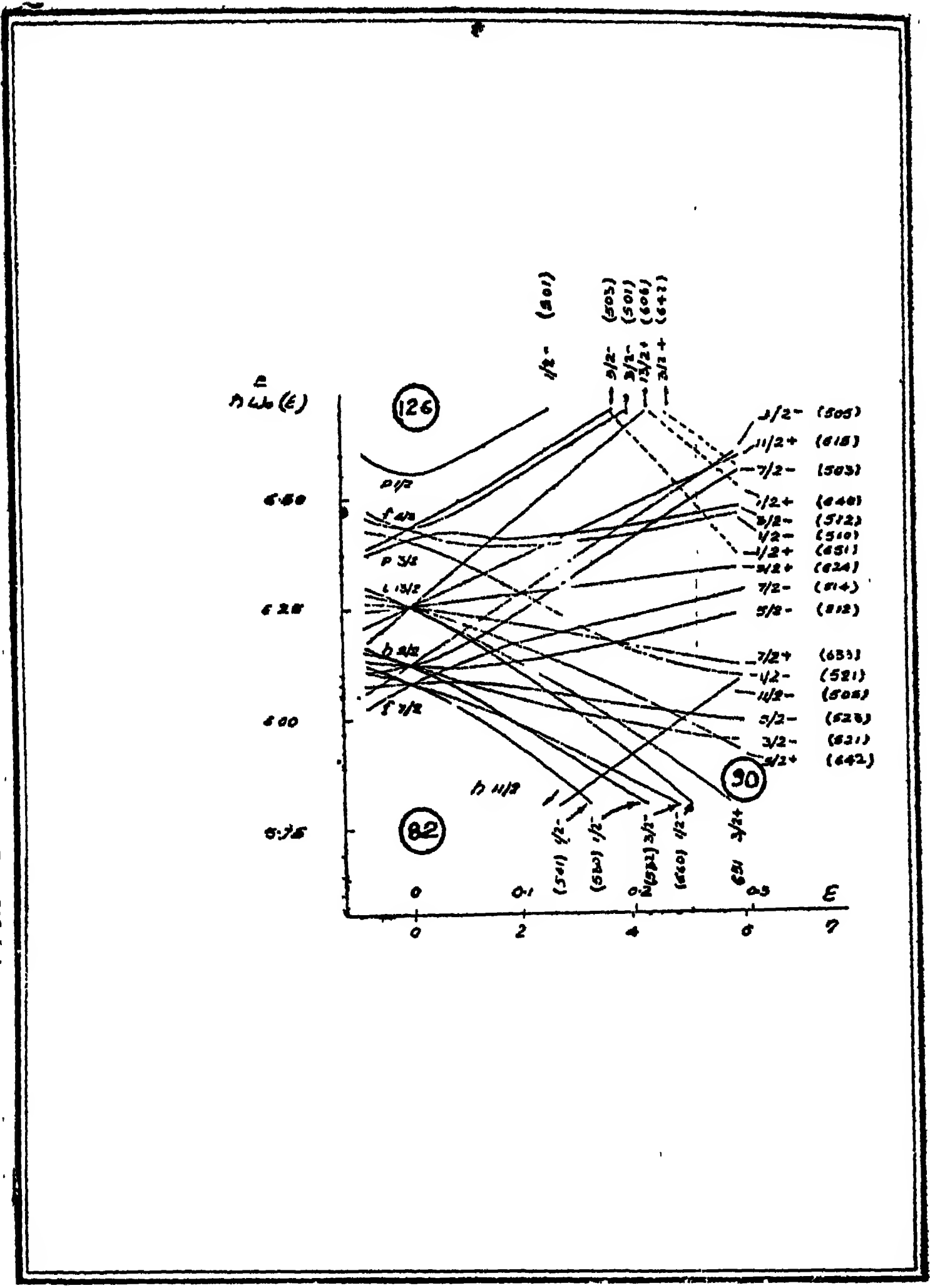


Figure 3.6
Single Particle Levels $50 < Z < 82$



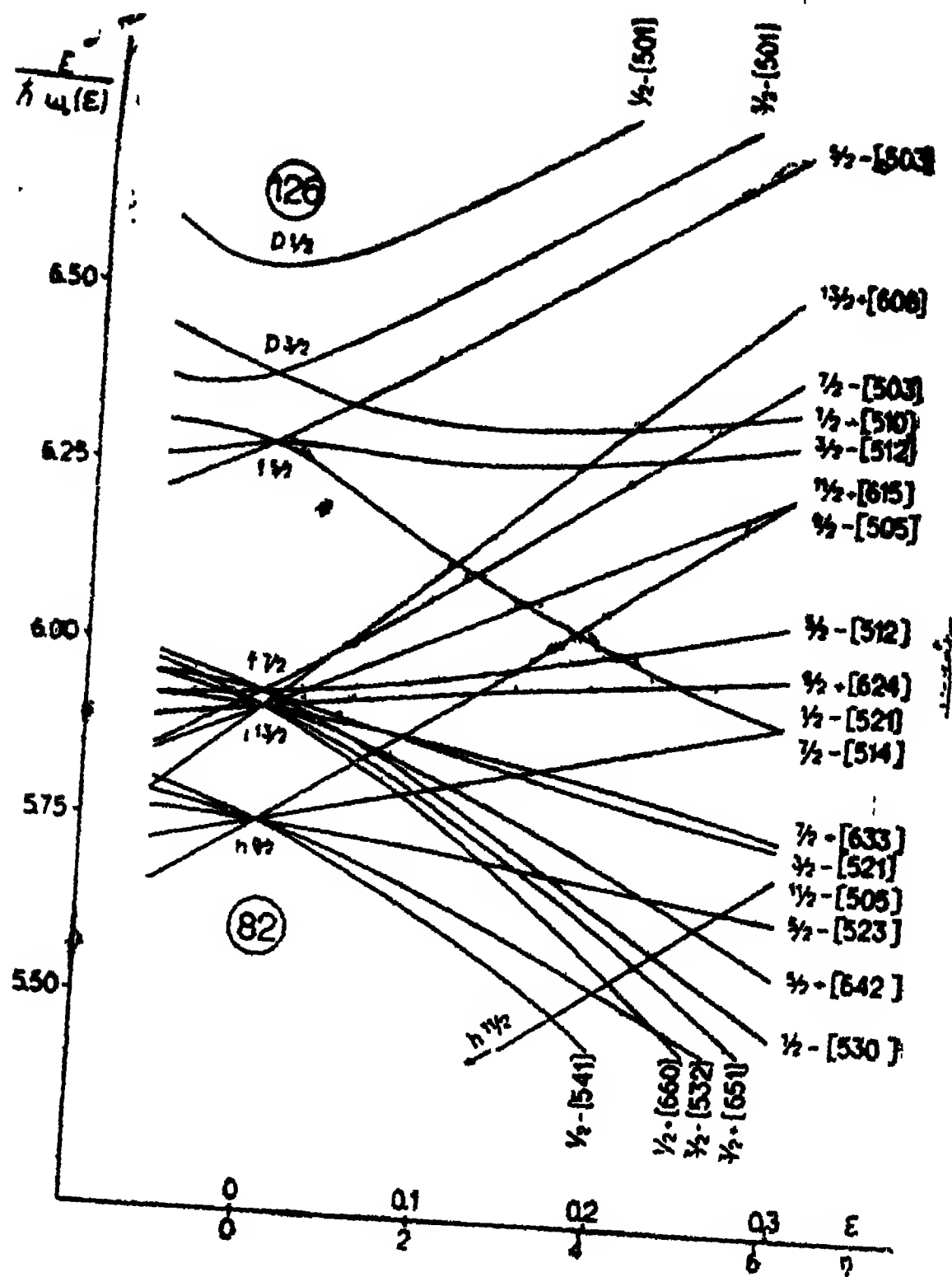


Figure 3.8
Odd-Proton Levels $Z > 82$

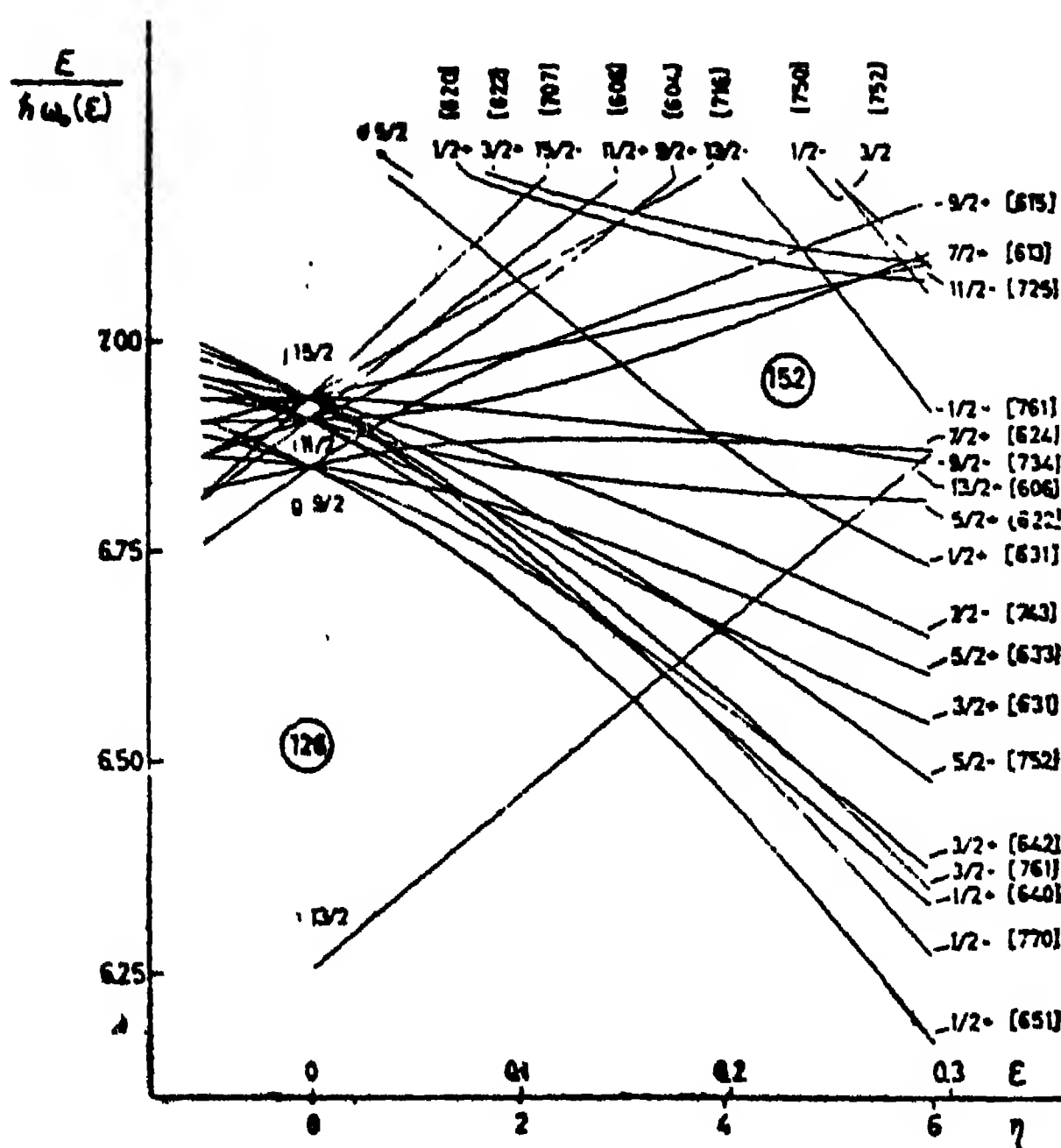


Figure 3.9

Odd-Neutron Levels for $N > 126$

The selection rules in terms of the asymptotic wave functions govern β and γ transitions as well as some special cases of anomalous conversion which are listed in references (25), (26), (27) and (28).

3.11 The Odd-A Nuclei

The odd-A deformed nuclei have properties which in many respects may be discussed in terms akin to those of odd-A spherical nuclei. There is however, an important difference, being associated with the fact of the degeneracy of the single particle spectrum which is lifted. As a consequence of this splitting the single particle density ρ becomes nearly uniform for the largest deformations. A consideration of moments and

transition rates leads to the occurrence of the main modification due to pairing in relation to extreme single particle model. The spin and parity of the quasi particle state are the same as those of the corresponding single particle state. Let there be no large fluctuation in level density. Then the single particle level associated with the lowest quasi-particle state of an odd system is the same as that occupied by the odd-particle diagrams from figures 3.6 to 9 can be applied as before with out any detailed consideration of pairing for the prediction of the ground state. Moreover the lowest excited states are the same as in the simple independent picture. The quasi-particle picture involves single particle levels with equidistant spacing resulting in too large a density near the ground state. In this picture, the mean distance between the ground state and the first excited state may be 50 keV whereas the empirical level distance is of the order of 150–200 keV. This discrepancy, however, can be diminished if a Poisson distribution of single particle states be assumed. Also this situation may to some extent be improved by the inclusion of blocking. It can be seen that there is a remarkable agreement between the empirical spins and parities of one quasi-particle states in odd-A nuclei in the regions I and II compared with one single particle orbitals. An inclusion of blocking to some extent may improve the situation. Moreover, the properties such as the decoupling factor and magnetic moments are not affected since the angular momenta of the two time-reversal orbitals cancel out whereas the electric moments are slightly modified.

In the case of pairing factors for gamma transitions, the one for $m\lambda$ with $\Delta s=0$, being of no change in the number of quasi-particles is negligible for low energy type. The transitions which are classified in terms of asymptotic selection rules as hindered or otherwise may be applied practically without modification. The pairing factor for the ϵ_λ may be of decisive importance although there may arise a confusion of the hindrance due to pairing with that arising out of other effects. In this way the large hindrance factor of the order of 10^{-4} – 10^{-8} met with for E1 transitions of low energy in the region of odd-A nuclei may be attributed to the intrinsic hindrance factor which may be expressed in terms of selection rules in the

asymptotic quantum numbers. In the case of low energy transitions the pairing factor which is nearly proportional to

$$\left[\frac{(\epsilon_1 - \lambda) + (\epsilon_2 - \lambda)}{2\Delta} \right]^{\frac{1}{2}}$$

may be expected to be next in importance. The specific correlation effect is responsible for the production of the giant resonant state and it subsequently exhausts most of the E1 oscillator strength. The E1 single particle transition probabilities calculated on the basis of wave functions yield⁽²⁹⁾ results which are of one, two or three orders of magnitude larger than those empirically obtained. Sometimes, they are said to likely be of the same order or sometimes even smaller by an order of magnitude^(30,31). The discrepancy could likely have been larger than that found experimentally in view of the latter hindrance factors. However, such a conclusion may not be justified owing to the uncertainty prevailing in the calculation of very small hindered matrix elements from the single particle wave function. The other $E\lambda$ as $M\lambda$ transitions follow qualitatively by the asymptotic selection rules. An average extra-hindrance of electric transition $E\lambda$ relative to the magnetic transitions $M\lambda$ due to pairing factor though expected, does not seem to have been established.

The classification of β transitions as hindered with respect to the asymptotic selection rules has been found successful⁽³²⁾ even without regard to pairing. Thus pairing does not seem to affect this classification. In odd-A spectra, three quasi-particle states are expected to occur in the spectra at an excitation energy of approximately 2Δ which are reported to have been experimentally observed in the spectra of Lu^{177} and Hf^{177} (Fig. 3.10).

3.12 Odd-odd Nuclei

The system of these nuclei is in many respects very similar to that of odd-A nuclei. Ignoring the residual forces, the total wave function is the product of the one quasi-particle proton and one quasi-particle neutron wave function. There

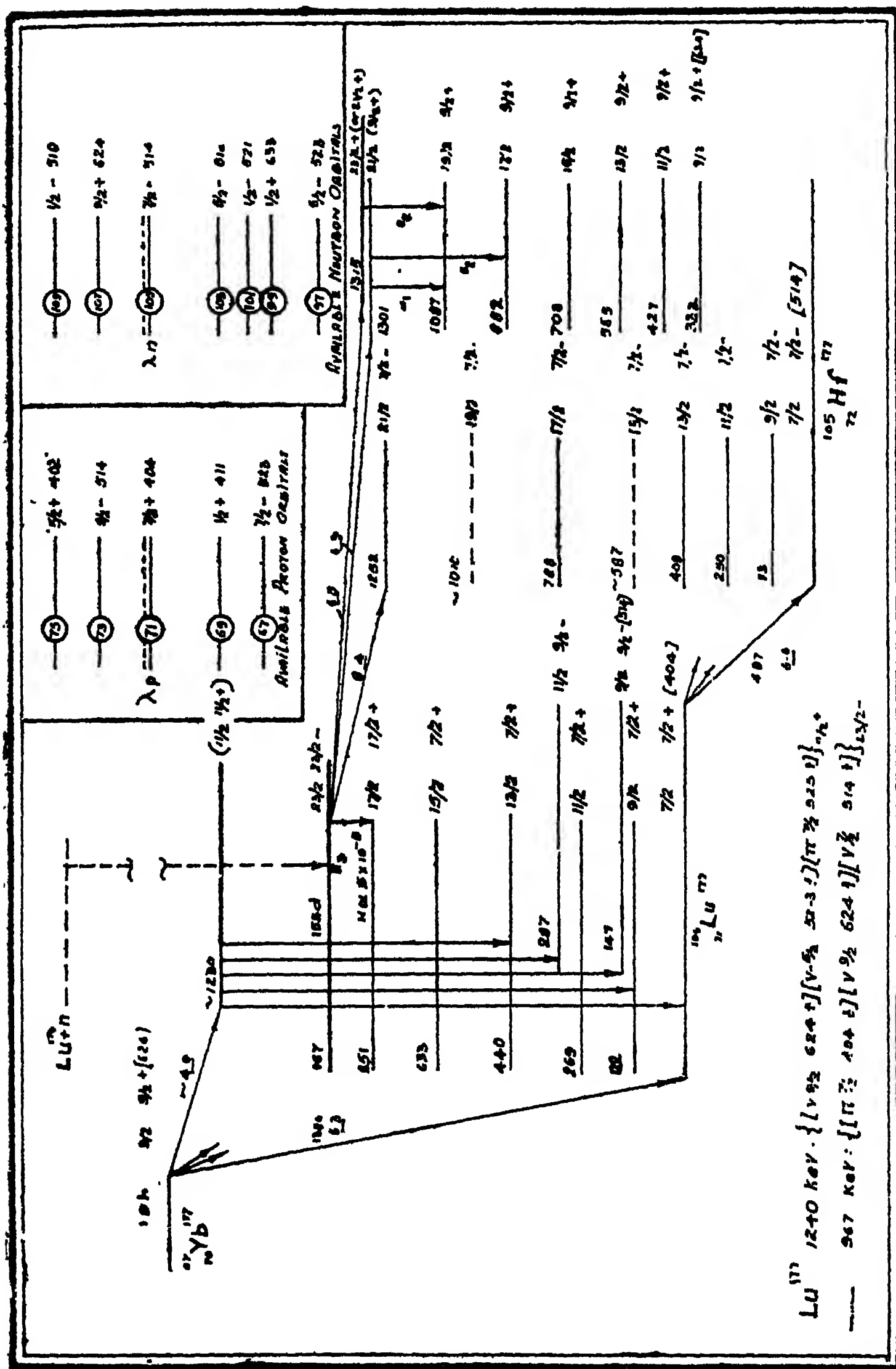


Figure 3.10
One and Three Quasi-particle States in ^{177}Lu and ^{177}Hf

can be two different intrinsic states $K = \Omega_p + \Omega_n$ and $K = |\Omega_p - \Omega_n|$ for each system (Ω_p, Ω_n) . In the case of $\Omega_p = \Omega_n$ and $K = 0$, symmetric and antisymmetric combinations exist given by $(X_{\Omega_p} X_{-\Omega_n} \mp X_{-\Omega_p} X_{\Omega_n})$ of which the first corresponds to $\gamma = +1$ and rotational band with $I = 0, 2, 4, \dots$ while the second $\gamma = -1$ and a rotational band with $I = 1, 3, 5, \dots$. The intrinsic energies of the $K = \Omega_p + \Omega_n$ are found to have been separated from the $K = \Omega_p - \Omega_n$ states by the residual interaction. In a few cases when both states are identified in the same nucleus, the magnitude of separation is usually of the order 100 keV for $A > 150$. A rule due to Gallagher and Moszkowski⁽³³⁾ has been deduced from the data of spins and parities which enunciates that the lowest of these states corresponds to parallel intrinsic pairs in the asymptotic representation. The rule holds for all empirically known cases save Ho^{166} in which case, which of the isomers is the ground is not definitely clear. In an interaction in a relative s -state which involves single odd neutron-odd proton, the parallel spins

alignment is preferable owing to a strong $\overset{\rightarrow}{\sigma}_1 \cdot \overset{\rightarrow}{\sigma}_2$ component in the nuclear force as in the deuteron. However, the odd-neutron and odd-proton states may usually correspond to the relative wave function quite different from that of the deuteron. Moreover the relative $n-p$ wave functions strongly vary from one odd-odd case to another. The validity of the rate-rule is mainly an effect of pair correlation⁽³⁴⁾. In the quasi-particle picture

only the $\overset{\rightarrow}{\sigma}_1 \cdot \overset{\rightarrow}{\sigma}_2$ part of the interaction continues to exist on account of different time reflection characteristics of the interaction operators. Moreover, the residual interactions are responsible for the splitting of the two $K = 0$ bands. Also the residual interactions account for the splitting of the two $K = 0$ bands based on the same intrinsic orbitals (but with) $\gamma = +1$ and even spins and $\gamma = -1$ and odd spins, the latter⁽³⁵⁾ having been studied in Ho^{166} , Am^{242} , Lu^{173} . The odd-neutron as well as odd-proton may be excited with increase in quasi-particle number. Analogously a great density of states at low energies is found to exist. The odd- A spectra are known in detail while corresponding spectra in odd-odd nuclei are by no means so. The assignments in the latter case are less unique than in those

in odd-A. Nevertheless they are consistent with the single configurations met with in odd-A nuclei.

3.13 Even-even Nuclei

All the particles filling paired orbits $(\Omega, -\Omega)$ form the constituents of the ground state in which the correlation between the pairs then is responsible for the low energy of the state and the occurrence of the energy gap in these even-even nuclei in contrast to the odd mass cases. Just above the gap occurs a great density of intrinsic states amounting to two quasi-neutron states. It may be actually expected to be larger than in the odd-odd case near the ground state. Residual interactions play even a more drastic role in the odd-odd case on account of the better overlap. The two quasi-particle states in even-even nuclei are difficult to classify. In several cases a certain measure of uncertainty prevails as to whether the lowest $K \neq 0$ states is of the broken neutron pair or that of the broken proton pair. It is, however, in many cases a unique assignment on the basis of a superallowed $\{\Delta\Sigma = 1, \Delta\Lambda = \Delta n_Z = 0\}$ β transition populating this state. The two neutron state [ν 523 5/2, 642 5/2] Dy^{162} populated by β decay, $\log ft = 4.6$, from the Ho^{162} 100 keV isomer [π 523 7/2, ν 642 5/2] is an illustrative example of the mentioned type of assignment. A classification of two quasi-particle states in region II has been attempted⁽³⁶⁾. The attempt involves some preliminary energy calculations with the inclusion of blocking, and without making any changes in particle number of fluctuations.

3.14 Collective Nuclear Vibrations in Deformed Nuclei

The description of collective nuclear vibration for spherical nuclei given in the preliminary sections is required to be slightly modified in order to enable it to be applicable to the nuclei of deformed type. On account of the deformed nature, the angular momentum λ of the excitation is no longer a constant of the motion but can be only its component on the symmetry axis for nuclei of axial symmetry. In the limit of small nuclear distortion, λ corresponds to the multipole order

and as such can be useful for classifications of the vibrations.. Moreover the parity of the phonon equals $(-)^{\lambda}$.

It is convenient to express the vibrational variables with respect to the axis system fixed in the distorted nucleus⁽³⁷⁾ being in this case be denoted as $\alpha_{\lambda\nu}$, inasmuch as the frequencies of vibration are greater than those of vibration. The vibrational modes corresponding to different ν values are not degenerate.. The non-collective intrinsic motion and the vibrational motion which are respectively characterized by the angular momentum component K_0 and the angular momentum ν couple in such a way that for a total angular momentum K , the relation that $K = |K_0 \pm \nu|$ holds so as to lead the adiabatic coupling scheme to $I \geq K$.

3.14(a) Vibrations of Quadrupole Type with β and γ Character

In the case of quadrupole vibration, α_{20} and $\alpha_{22} = \alpha_{2, -2}$ are nonvanishing. Now let the mass parameters β_{20} and β_{22} a force parameters C_{20} and C_{22} be defined with respect to the variables α_{20} and α_{22} . The latter may be expressed in terms of the β and γ the elongation parameter and the axial-symmetry angle γ as

$$\alpha_{20} = \beta \cos \gamma \quad \dots(3.69)$$

$$\alpha_{22} = \alpha_{2,-2} = \sqrt{1/2} \beta \sin \gamma \quad \dots(3.70)$$

where β , the surface parameter is related to the potential parameter ε from the relations given by the expressions (3.46) and (3.47) so that the potential parameter ε may be expressed to the first order as

$$\varepsilon \simeq \frac{3}{4} \sqrt{\frac{5}{\pi}} \beta$$

The vibrations associated with α_{20} and those with α_{22} are termed as β and γ vibrations respectively. In the case of equilibrium axial symmetry and small axial symmetry angle γ , it can be said $\alpha_{20} \simeq \beta$ and $\alpha_{22} \simeq \beta\gamma/\sqrt{2}$. Of these modes the γ -vibrational ones are so far empirically identified. The B (E2)

values may be found to be of the order of 3 to 6 times the single proton estimate. Also they exhaust a significant fraction of the E2 oscillator sum rule. Further more, the moment of inertia of the rotational bands resulting from the β and γ vibrations, unlike that of the octupole bands is nearly the same as that of the corresponding ground state band.

The members of the $K = 2^+$ and $K = 0$ bands are a first approximation branched to the members of the ground state band in even-even nuclei obeying the rules enunciated in (3.46). There are however, significant deviations. In the case of γ vibrations, this deviation can be analyzed in terms of the coupling parameter Z_2 which measures the rotation vibration interaction. A coupling to lowest order in the $K = 0$ is given rise by the term H^1_{rot} while the γ vibrational band is coupled directly to the ground state band by the term H^3_{rot} . The bands in the second order, however, are mixed by the H^2_{rot} . Also H^3_{rot} and H^1_{rot} may be considered to be representing the second order effects of H^2_{rot} . The second order couplings with the vibrational states, unlike the couplings to other $K = 0^+$ or $K = 2^+$ states are of importance for the F_0 transition rate inasmuch as these mixings are connected with the collective matrix elements proportional to Q_0 . In the case of energy depression of the ground state rotational band proportional to $I(I+1)^2$, the coupling to the vibrational states could be concluded to account for about 5 - 10% of this depression the rest being ascribed to Coriolis couplings to the states other than vibrational ones. The $B(E2)$ values can be expressed in terms of the Z -parameters as

$$B(E2, I_i K=0 \rightarrow I_f K=\nu) = B(E2)_0 [1 - z_\nu f_\nu(I_i I_f)]^2 \dots (3.71)$$

where the definitions are applied which are given by

$$Z_0 = - \sqrt{\frac{5}{16\pi}} \epsilon_0 Q_0 < 0' | \mathcal{M}'(E2, 0) | 0 >^{-1}$$

$$Z_2 = - \sqrt{\frac{15}{2\pi}} \epsilon_2 Q_0 < 2 | \mathcal{M}'(E2, 2) | 0 >^{-1} \dots (3.72)$$

and where the reduced mixing amplitude ϵ_ν being given in

terms of h_0 and h_2 as

$$\epsilon_\nu = \frac{\langle \nu | h_\nu | 0 \rangle}{\epsilon_\nu - \epsilon_0} \quad \dots(3.73)$$

The quantity $f_\nu(I_i I_f)$ for $\nu = 0$ is expressed as

$$f_0(I_i I_f) = I_f(I_f + 1) - I_i(I_i + 1) \quad \dots(3.74)$$

and for $\nu = 2$ it may be written as

$$f_2(I_i I_f) = 1 + [I_i(I_i + 1)]^{1/2} \langle I_i 211 | I_i 2 \rangle \langle I_i 202 | I_i 2 \rangle^{-1} \dots(3.75)$$

The rotational energies of the ground state band due to the rotation-vibration interaction in their estimation require correction to be made with the correction terms δE_0 and δE_2 which are expressed as

$$\begin{aligned} \delta E_0 &= -\epsilon_0^2 (\epsilon'_0 - \epsilon_0) I(I + 1)^2 \\ \delta E_2 &= -2\epsilon_2^2 (\epsilon_2 - \epsilon_0) \{ I^2(I + 1)^2 - 2I(I + 1) \} \quad \dots(3.76) \end{aligned}$$

It may be stated that β and γ vibrational states in the spectra of odd-A nuclei are identified in a number of cases such as the spectrum of ${}_{67}\text{Ho}^{165}$ and that of ${}_{65}\text{Tb}^{159}$.

3. 14(b) Octupole Vibrations

The octupole states in spherical nuclei or in deformed nuclei, are expected to be split into components with $|\nu| = 0, 1, 2, 3$ of which the $K = 0$ — components with $I = 1-, 3-, 5- \dots$ rotational members, are found to exist in a large number of nuclei. The $K = 0$ — bands are found at low energies mostly in the neighbourhood $A = 224$. At comparatively higher energies the $K = 1$ — and 2 — states are in a few cases have been observed. The occurrence of large S — values of the $K = 0$ bands indicates the presence of strongly Coriolis coupled $K = 1$ member in close vicinity with respect to energy. The odd members of the $K = 0$ — band are expected to be displaced upwards relative to the even members on account of the Coriolis coupling to the $K = 0$ — band.

3.15 Deviation from Axial Symmetry

An investigation of γ vibrational states involves a consideration of momentary deviations from axial symmetry. An assumption of an equilibrium axial symmetry seems to be adequate to describe the main body of deformed nuclei. It looks as though there are indications that in the transition regions between deformed and spherical nuclei, it is appropriate to discuss the possible effects of stable equilibrium non-axial distortions^(38,39). Then the complete rotor Hamiltonian may be considered as given by

$$H_{\text{rot}} = \frac{\hbar^2}{2\mathcal{J}_1} I_1^2 + \frac{\hbar^2}{2\mathcal{J}_2} I_2^2 + \frac{\hbar^2}{2\mathcal{J}_3} I_3^2 \quad \dots(3.77)$$

Employing the representation (IKM) the states may conveniently be classified if $\mathcal{J}_1 - \mathcal{J}_2$ be small compared to $\mathcal{J}$, the $\mathcal{J}$ being defined by

$$\mathcal{J}^{-1} = \frac{1}{2} (\mathcal{J}_1^{-1} + \mathcal{J}_2^{-1})$$

If the deviation from axial symmetry be small. A small $\mathcal{J}_1 - \mathcal{J}_2$ may be expected although it may be expected even for other reasons. The lowest order energy for a rotor in such a case takes the form given by

$$E_{\text{rot}}^{(0)} = \frac{\hbar^2}{2\mathcal{J}} [I(I+1) - K^2] + \frac{\hbar^2}{2\mathcal{J}_3} K^2 \quad \dots(3.78)$$

In the derivation of this expression, the non-diagonal term is ignored which is given by

$$H_{\text{int}} = \frac{1}{2} \left(\frac{\hbar^2}{2\mathcal{J}_1} - \frac{\hbar^2}{2\mathcal{J}_2} \right) (I_1^2 - I_2^2) \quad \dots(3.79)$$

the couplings with $\Delta K = 2$ being given by this latter one, an inclusion of which in perturbation approximation leads to a modification of the ground state band by $I^2 (I+1)^2$ dependent term following (3.76). The observed accuracy of the energy dependence of the ground state band may be taken as a measure of deviation from axial symmetry likely to occur. If the deviations from axial symmetry be not small, K cannot be even approximately constant of the motion. In such a case a complete diagonalization of (3.76) gives the spectra. It is then

proper to consider all the three of $\mathcal{J}_1$, $\mathcal{J}_2$ and $\mathcal{J}_3$ as free parameters finally to be computed from data fitting⁽⁴⁰⁾. It is however, required to add β and γ vibrational ones to the rotational degrees of freedom so as to make a detailed comparison with experiments becomes intelligible.

Thus a very large number of parameters becomes involved in the problem. The effect of one degree of freedom at a time may be understood by considering the relation given by the expression (3.78) as illustrated in Fig. 3.11 which conse-

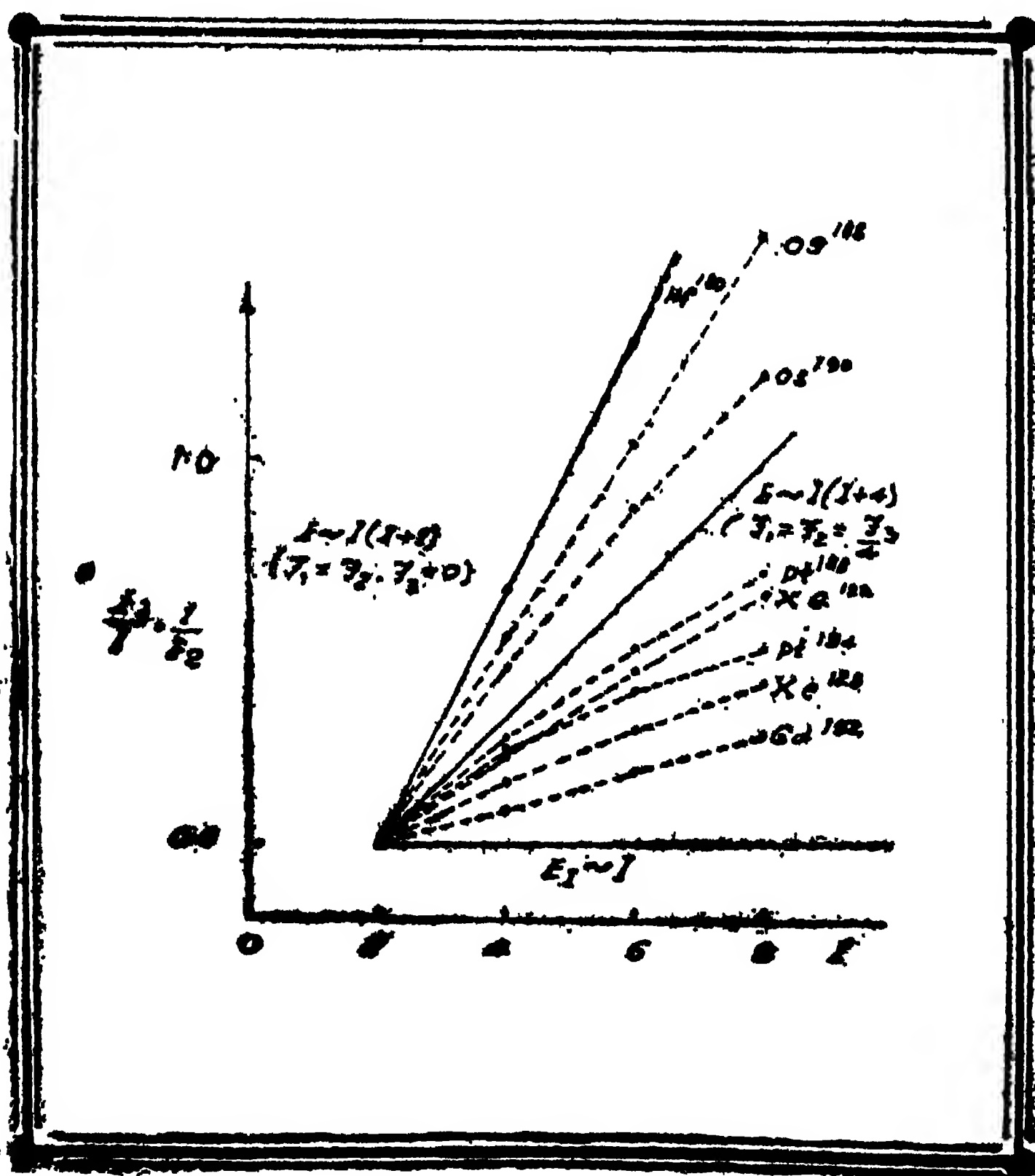


Figure 3.11

Lowest States of Given I and Positive Parity
in the even-even Nuclei

quently corresponding to $\mathcal{J}_1 = \mathcal{J}_2$ and $\mathcal{J}_3 \neq 0$. Then the angular momentum component K remains good quantum number and the case does not imply axial symmetry but on the other hand rotation around the 3-axis is assumed to be quantum mechanically definable. The condition $\mathcal{J}_1 = \mathcal{J}_2$ can be realized by the highly asymmetrical situation $\gamma = 30^\circ$ and $\gamma' = 0$. On the basis of hydrodynamical hypothesis in which case also $\mathcal{J}_3$ is fixed as being equal to $4\mathcal{J}_1$. Let rotor levels be studied as functions $\mathcal{J}_3/\mathcal{J}_1$ as represented in Fig. 3.12. In the case of small values of this ratio, the $K=0$ band members of the $K=0$ band are energy favoured compared to other levels of the same I but different K . If $\mathcal{J}_3 = \mathcal{J}_1 = \mathcal{J}_2$, the states of given I will all be degenerate in K whereas out of the states with given I -values those with $K=1$ are lowest in energy in the case of $\mathcal{J}_3 > \mathcal{J}$. These $\mathcal{J} = K$ states follow the single energy rule⁽⁴¹⁾ given by

$$E_I(I, K) = \frac{\hbar^2}{2\mathcal{J}_3} I \left(I + \frac{\mathcal{J}_3}{\mathcal{J}} \right) \quad \dots(3.80)$$

The empirical values of E_I/I in units of E_2 with such characteristics that they are lowest occurring ones which correspond to a given I -value and positive parity are plotted and in the plot shown in Fig. 3.11 a number of representative type of even-even nuclei Hf^{180} , Os^{188} , Os^{190} , Pt^{198} , Xe^{122} and Gd^{162} are chosen. The plot shows in the form of solid lines the energy ratios $E_I/I)E_2^{-1}$ for 1) the $\mathcal{J}_3=0$, $K=0$ rotational band 2) the $I=K$ and $\gamma=30^\circ$ in the hydrodynamical model 3) the pure harmonic vibrator.

The empirical states on the above mentioned basis are so well-described by a second order polynomial in $\mathcal{J}$ as given in (3.80). This behaviour, so far has been observed only for small regions of transition nuclei, this feature, however, by no means being common to all transition regions. The regularities which can be observed in Fig. 3.12 tend to suggest simple rotational picture. Even so it may be noted that important degrees of freedom are ignored, that the hypothesis $\mathcal{J}_1 = \mathcal{J}_2$ does not seem to be natural and also ignoring the vibrational degrees of freedom could not be serious.

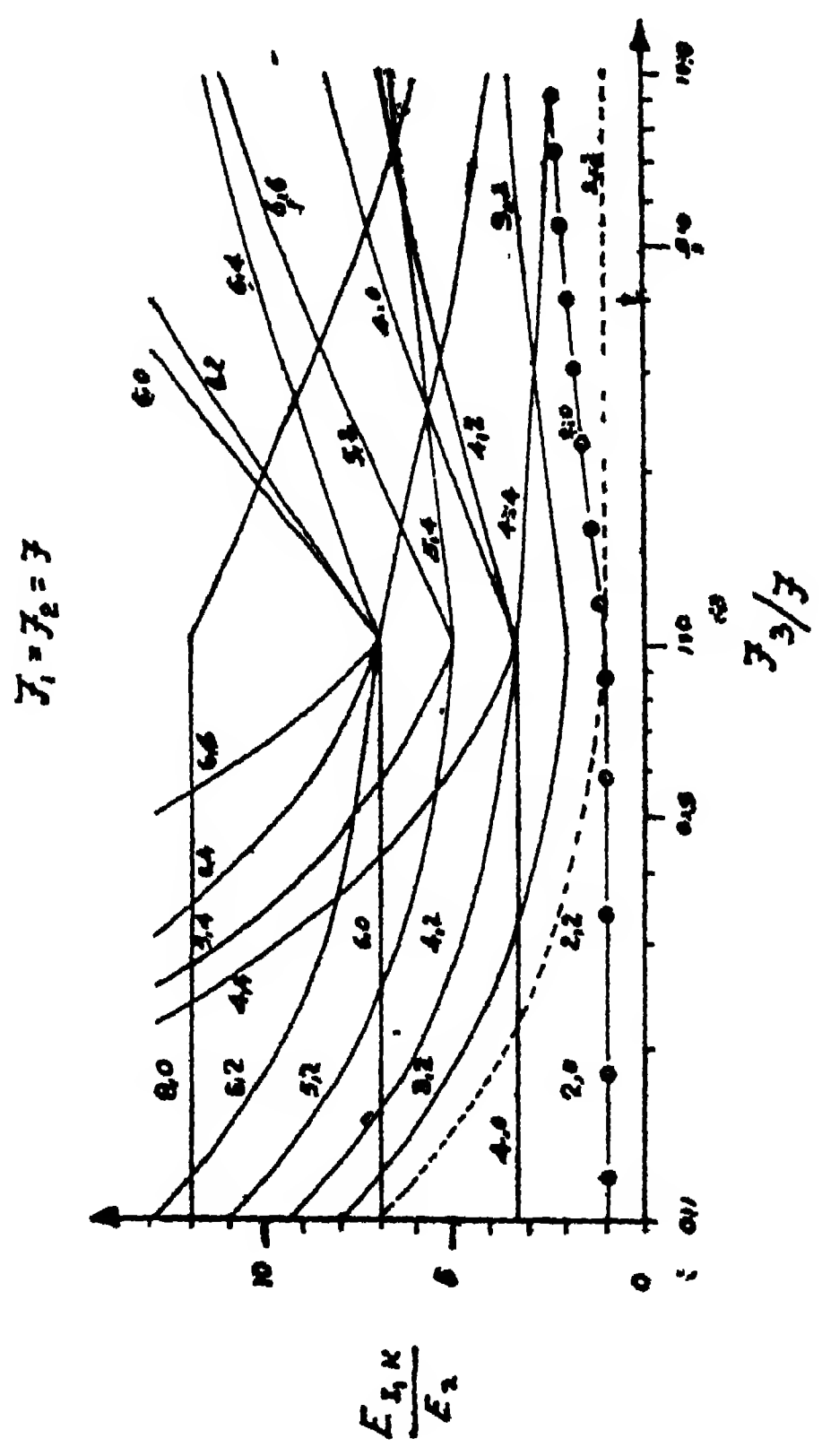


Figure 3.12
Energy Levels of the Asymmetric Rotor with $J_1 = J_2 = J_3$

The analysis of the only pure rotor requires similar several such plots as (3.11) valid for $\mathcal{J}_1/\mathcal{J}_2$ other than one. In the case of $\mathcal{J}_1 \neq \mathcal{J}_2$, there can be no analytical expressions for E_{rot} . However, it may be determined by resorting to numerical evaluations⁽⁴²⁾. Moreover, it is, as already mentioned, required to add the additional degrees of freedom which complicates the picture.

The three free $\mathcal{J}_K$ parameter including the case that these parameters are related via the hydrodynamical model have been computed for even-even nuclei in the $A \simeq 190$ region. These numerical calculations involve the rotation vibration interaction⁽⁴³⁾. The fitting of a large number of data in terms of relatively few parameters seems to go beyond the expected on the basis of pure parameter fitting. The treatment of the degrees of freedom of the axially asymmetric even-even rotor encounters great complication which is additively enhanced when the intrinsic structure as modified by the axial asymmetry has to be considered for the description of the low lying states of an odd- A nucleus. An average potential similar to that of (3.64) but with $\omega_x \neq \omega_y$ has been employed for computing intrinsic wave functions^(44, 45). The intrinsic angular momentum component Ω as well as the total angular momentum component K are not constant of the motion. Hence the total wave function cannot be of the simple type expressed by (3.25). Nevertheless it may be expressed in terms of sums over components of given Ω and K . Consequently the simple separation between intrinsic and rotational degrees of freedom is largely annihilated.

In the case of nuclei in the region of $A \simeq 190$ where the axially symmetric description of (3.24) seems to fail, attempts have been made⁽⁴⁶⁾ thereby indicating that it is possible to fit at least the lowest levels of the energy spectrum such as that of Pt^{195} without vibrations taking into account. Moreover, hydrodynamical assumptions are required to be employed in these calculations to relate the ratios of the moment of inertia. Nevertheless the level fit is not unique. There is a chance of different sets of β^- and γ parameters and particle states. Also it becomes impossible to fit at the same time all of the observed

transition probabilities even with one or two orders of magnitude with these parameter sets thereby bringing out the necessity of introducing the vibrational degrees of freedom in order to explain the large strength of some E2 transitions from the ground state.

3.16 Collective Motion and Collective Parameters

The collective rotational motion: The collective rotational motion has already been dealt with in terms of phenomenological collective parameter, $\mathcal{J}$ and g_R . Also B and C are introduced largely as phenomenological parameters in the description of the collective vibration. Moreover, the basic relation of the single particle degree of freedom was strongly but qualitatively employed with those of the collective vibrations.

Now let the Hamiltonian of the entire system in the form of a power series expansion be considered in the total angular momentum $\mathcal{J}$. This procedure can be suitable if the rotational motion is slow compared with the single particle motion. The discussion of the intrinsic field and the pairing concerns the independent part of this expression. Denoting this part of the Hamiltonian by H_0 , the I-dependent term may be calculated by considering⁽⁴⁷⁾

$$H' = H_0 - \mu \hbar J_x \quad \dots(3.81)$$

Where μ , the Langrangian multiplier is so chosen as to obtain a specified value I_x of the angular momentum component J_x along the x-axis, x being the axis of rotation which is fixed at right angles to the Nuclear symmetry axis

$$\langle J_x \rangle = I_x \quad \dots(3.82)$$

The solution to (3.81) is obtained by a variational method and the quantity H' is made stationary with respect to any variational parameter, such as I_x . The condition $\delta \langle H' \rangle / \delta I_x = 0$ leads to

$$\mu = \hbar^{-1} \frac{\delta \langle H_0 \rangle}{\delta I_x} \equiv \omega_x \quad \dots(3.83)$$

which forms the natural definition of the angular frequency ω_x of the collective rotation: the derivative of the total energy with respect to the angular momentum.

The quantity H' may be treated as the Hamiltonian referring to the rotated system, the term $-\hbar \omega_x J_x$ representing the Coriolis and centrifugal forces operating in the moving system. In this connection, H_0 is implicitly assumed to be independent of the rotational frequency ω_x which amounts to the rotating field being the same as the static field. Then let these be expanding in solutions to H_0 and let the added term be treated as a perturbation so that to the first order

$$| > = | 0 > + \hbar \omega_x \sum_i \frac{\langle i | J_x | 0 \rangle}{\epsilon_i - \epsilon_0} | i > \dots (3.84)$$

The expression (3.82) gives the moment of inertia $\mathcal{J}_x$ defined as $\hbar I_x / \omega_x$ in the form written as

$$\mathcal{J}_x = 2 \hbar^2 \sum_i \frac{|\langle i | J_x | 0 \rangle|^2}{\epsilon_i - \epsilon_0} \dots (3.85)$$

This derivation can be understood by pointing out the choice of H_0 in (3.81) is of a particular selection of a reference frame in which the quasi-particles are known to be moving independently. Its wave function and the aligned wave function Φ_{aligned} of the form⁽⁴⁸⁾

$$\Phi_{\text{aligned}} = \begin{vmatrix} \Psi_{jj} (1) & \Psi_{j-j} (1) & \Psi_{jj-1} (1) \\ \Psi_{jj} (2) & \Psi_{j-j} (2) & \Psi_{jj-1} (2) \\ \Psi_{jj} (3) & \Psi_{j-j} (3) & \Psi_{jj-1} (3) \end{vmatrix} \dots (3.86)$$

will be analogous to the unperturbed state $| 0 >$ of (3.84). The effect of the coordinate frame is ignored and total angular momentum is not a constant of the motion since these two features are intimately related to each other. The auxiliary condition accounted for in (3.81) by the introduction of Lagrangian multiplier ensures that the wave function corresponds to a definite angular momentum value. Thus the transformation is carried out from the particular reference frame H_0 to the laboratory coordinates. The above expression (3.85) was

derived⁽⁴⁹⁾ on the basis of 'Cranking model' according to which the nucleonic motion in a field that is cranked around externally is considered. The extra energy that is necessary so that the particles should follow the field adiabatically to sustain the field is calculated. Because of this mode of derivation, this expression is termed as "Cranking formula". In the case of a pure isotropic harmonic oscillator potential at equilibrium, the rigid value of the moment of inertia is exactly obtained as already proved⁽⁵⁰⁾

It can be seen that the empirical value of $\mathcal{J}$ for even-even nuclei in region II is only 20–50% of $\mathcal{J}_{\text{rig}}$. It was shown⁽⁵¹⁾ that the disparity was reduced by the inclusion of the residual interactions. The theoretical picture was based on the schematic 'two-nucleon model'. A similar analysis on the basis of a more detailed nuclear model, the most important part of the residual interaction in deformed nuclei may be accounted for by the pairing interaction. The above general expression (3.85) in the quasi-particle approximation may in terms of the single particle elements $\langle \nu' | j_x | \nu \rangle$, the diffusion factors U_ν and V_ν and the quasi-particle energies may be written as

$$\begin{aligned} \mathcal{J}_x (\text{even-even}) &= \\ &= 2 \hbar^2 \sum_{\Omega \nu > 0} \frac{|\langle \nu' | j_x | \nu \rangle|^2}{E_{\nu'} + E_\nu} (U_\nu V_{\nu'} - V_\nu U_{\nu'})^2 \end{aligned} \quad \dots(3.87)$$

Now it may be mentioned that the energy associated with the matrix element $\langle \nu' | j_x | \nu \rangle$ is $(\epsilon_{\nu'} - \epsilon_\nu)$ amounting to just the difference between the single-particle energies. The expression for the moment of inertia is modified by the pair correlation by the effect of the energy difference between the even-even ground state and the excited configuration which is very much increased on account of the odd particles of the broken pair preventing the correlation pairs from using the single particle states ν and ν' . Hence the energy is always larger than 2Δ . Moreover, as a consequence of $(U_\nu V_{\nu'} - V_\nu U_{\nu'})^2$ the state ν is required to be occupied by a pair V_ν and the

state ν' be occupied by $(U_{\nu'})$. The operator $J_x = -\sum_i j_x^{(i)}$ then scatters a particle from ν into ν' . The reverse situation that ν is occupied by a pair and the ν' is possible unoccupied. The two processes actually interfere. The minus sign is directly connected to the time reflection properties of J_x operator. These effects reduce the value of $\mathcal{J}$ by about the same factor. The theoretical values are found to be systematically smaller than the empirically obtained ones by an order of 5–20%. If the decrease in Δ be by 10% and increase in the deformation parameter ε by 10%, the discrepancy will be removed. There is, however, corresponding amount of uncertainty. Even so, the systematic discrepancy seems significant, thereby confirming the significance of the deviation. The discrepancy thus resulting can be removed if the blocking be carefully taken into account. It cannot be ascertained how much of the deviation can be ascribed to the excessive simplification of the residual interaction by the use of pairing force there by necessitating further enquiry into the matter.

The moment of inertia, $\mathcal{J}$ is a measure of the mass transported in collective rotation flow while the collective Gyromagnetic ratio g_R is a measure of the magnetic properties of this collective flow. On the basis of an expansion of (3.84) the expression for g_R which is similar in structure to (3.87) may be derived. A comparison of theoretical values of g_R for even-even nuclei⁽⁵²⁾ with the experimental data shows that within the limits of the approximation that the spin contribution can be ignored, the mentioned g_R may be expressed as

$$g_R = \frac{\mathcal{J}_p}{\mathcal{J}_n + \mathcal{J}_p} \quad \dots(3.88)$$

This ratio comes out smaller than Z/A so that Δp empirically turns out to be larger than Δn . The values of odd- A moments of inertia are found to be larger than the corresponding even-even values while those of odd-odd are even larger than those of odd- A . The increase in $\mathcal{J}$ can be ascribed to the one-quasi-particle state being directly connected with another one-quasi-particle state by the J_x operator so that no room is given for the break of additional pairs. Hence the associated energy

denominator $(\epsilon_{\nu'} - \epsilon_{\nu})$ is small. The mentioned contribution compensates the loss sustained inasmuch as the level occupied by the odd particle is by no means available for the even contribution. The odd particles especially in the orbitals from $h_{11/2}$, $i_{11/2}$ and $j_{15/2}$ associated with large angular momentum matrix elements to near lying states of the same sub-shell have very larger contributions. Let the excess in the moment of inertia of the odd-A nuclei be denoted by $\delta \mathcal{J}$. Then the largest of $\delta \mathcal{J}$ is associated with the orbitals [642 5/2] [633 7/2] and [624 9/2] originating from $\mathcal{J}_{13/2}$. In nuclear spectroscopy in this region of nuclei the values of the moment of inertia offer most reliable clues for the identification of orbitals.

3.17 The Collective Vibrational Motion

The collective vibrational excitation may be viewed as a coherent superposition of single particle excitations. Such a view provides a qualitative but useful picture of the vibrational excitations. Moreover, this picture, usually termed as 'microscopic' can be employed in order to estimate the collective vibrational parameters. This description forms a major improvement over the hydrodynamic estimates. As a preliminary step towards for a further understanding, consider nuclei near doubly closed shells so that the short range correlation can be seen as likely the most important residual effect which can be introduced by the quasi-particle transformation. If H_0 be the Hamiltonian involving the nuclear potential and the pairing force, the quasi-particle transformation may assume the form given by

$$H_0 = \sum_{\nu} E_{\nu} (a_{\nu}^+ a_{\nu} + a_{\nu}^- a_{\nu}^-) + \text{Const} + H_{\text{int.}} \quad \dots(3.89)$$

Let the term $H_{\text{int.}}$, representing interaction between quasi-particle be treated as negligible. The two quasi-particle states $a_{\nu}^+ a_{\nu'}^+ | \tilde{0} \rangle$ is associated with an excitation energy $E_{\nu} + E_{\nu'}$ relative to the ground state $| \tilde{0} \rangle$. The remaining long range part of the correlation, on expansion in terms of different multipoles yield the spin independent components of such an expansion. Also it is maintained by the results of detailed calculations

that each component is associated with the collective state of the same multipole character. The quadrupole part is especially important in the elucidation of the systematic occurrence of low lying 2^+ states in all even-even spherical nuclei. These 2^+ states have collective character as is evident from their E2 strengths. The octupole component in this expansion is likewise expected to be associated with the collective 3-states as can be observed in spherical nuclei and the $K=0$ -states in the deformed nuclei especially $A=225$.

A restriction of the view only to the quadrupole term of the multipole expansion given by $V(\vec{r}_1 - \vec{r}_2) = \sum_K (r_1 r_2) P_K \cos(\theta_{12})$ made for definiteness enables an easy generalization so that in terms of second quantization, the quadrupole term can be written as

$$H_Q = -\frac{1}{2} K \sum_{\mu} \sum_{\nu \nu_1} \sum_{\nu_1' \nu_1} \langle \nu | r^2 Y_{2\mu} | \nu' \rangle \langle \nu_1 | r^2 Y_{2\mu}^+ | \nu_1' \rangle a_{\nu}^+ a_{\nu'} a_{\nu_1'}^+ a_{\nu_1} \dots (3.90)$$

In the above expression, a specific simple radial dependence of H_Q is assumed. Expressing H_Q in terms of the a_{ν} and $a_{\nu'}^+$ operations gives terms⁽⁵³⁾ of the form written as

$$H_Q = H_{00}^Q + H_{11}^Q + H_{20}^Q + H_{22}^Q + H_{31}^Q + H_{40}^Q \dots (3.91)$$

the operators being ordered with the creation operators to the left of the annihilation operators and the indices referring to the number of a_{ν}^+ and a_{ν} operators in each term. The equation (3.91) shows that in the even-even case H_Q couples the two quasi-particle states with each other, the two quasi-particle states with the four quasi-particle states etc. It looks as if H_Q could be treated as a perturbation term. This procedure, however, by experience is found to fail⁽⁵⁴⁾ even for a small number of particles or holes outside of closed shells. The particular 2^+ states with strongly collective properties separate from all the other excited quasi-particle states. In this case of perturbation expansion has a poor convergence.

The approach which has then be made is one in which the two particle interaction is approximated with a quadrupole.

field. The relation given by (3.82) is expressed as a sum of single particle operator $Q_{2\mu}$ coupled to the average field $\bar{Q}_{2\mu}$. Thus in the generator Hamiltonian a term is introduced which is of the type given by

$$\hat{H}^Q = -K \sum_{\mu} Q_{2\mu} \bar{Q}_{2\mu}^* \quad \dots(3.92)$$

with

$$\bar{Q}_{2\mu} = \left\langle \sum_i \left(\frac{4\pi}{5} \right)^{\frac{1}{2}} r_i^2 Y_{2\mu}(\theta_i) \right\rangle \quad \dots (3.93)$$

It may be noted that the factor $\frac{1}{2}$ in the generator Hamiltonian vanishes by the application of this Hartree-Fock method.

The operator $Q_{2\mu}$ in terms of quasi-particle operator includes terms proportional to a^+a , a^+a^+ , aa in addition to a -independent terms.

The coupling constant K may be written as

$$K \simeq M\omega_0^2 \frac{1}{\sum_i \langle r_i^2 \rangle}$$

The equation (3.92) provided only the vibrational motion is slow compared with the relevant single particle frequencies. A certain self-sufficiency between the shape of the matter distribution and that of the field is assured if

$$\langle \Psi | Q_{2\mu} | \Psi \rangle = \bar{Q}_{2\mu} \equiv \alpha_{2\mu} \quad \dots(3.94)$$

for each component. Henceforth, the index μ is dropped for having simplicity in this schematic presentation. The quantity α defined in (3.94) must not be confounded with the quasi-particle creation and destruction operators a^+ , and a . It can be considered as the collective coordinate describing the changes in the shape of the nucleus as a whole. The total energy as a function of α can be investigated by studying the K -variation with α of the total nucleonic wave function $\phi(\alpha)$ and with the knowledge of $\phi\alpha$.

The $\phi(a)$ can be found by considering the Hamiltonian

$$H' = H_0 - (Ka + \mu) Q \quad \dots(3.95)$$

μ being a Lagrangian multiplier. Let the components of a and Q be dropped. Then the lowest order in a may be obtained so that

$$\phi(a) = |0\rangle + \sum_i (\mu + ka) \frac{\langle i | Q | 0 \rangle}{\epsilon_i - \epsilon_0} |i\rangle \quad \dots(3.96)$$

where μ being determined from the auxiliary condition given by (3.94), ϵ_0 and ϵ_i are the total energies of the given state $|0\rangle$ and the intermediate two-quasi particle state $|i\rangle$ in the even case for which

$$\epsilon_i - \epsilon_0 = E_\nu + E_{\nu'}$$

The matrix element $\langle i | Q | 0 \rangle$ likewise contains a (U, V) -factor in addition to a single particle matrix element.

The potential and kinetic energy in powers of powers of a and $\dot{a}$ may be expanded. The terms of lowest order under the condition of time reversal invariance are of the form written as

$$H_{\text{vib}} = \frac{1}{2} B |\dot{a}|^2 + \frac{1}{2} C |a|^2 \quad \dots(3.97)$$

If the deviation from spherical symmetry or in the deformed case the deviation from equilibrium be small which amounts that a is small, the harmonic approximation in the case of H_{vib} can be satisfactory. Also its harmonic approximation can be satisfactory if its vibrational frequency is low.

Then a treatment by the procedure of the second order quantization the collective vibration Hamiltonian given by (3.97) leads to the frequency ω given by

$$\omega = \left(\frac{C}{B} \right)^{\frac{1}{2}} \quad \dots(3.98)$$

Where B is the mass parameter and C the restoring parameters these features being already given in the expression (3.5). In

this connection it will be found that the mass parameter B and the restoring or the potential energy parameter C can be computed in terms of the Hamiltonian (3.95) and the wave function (3.96). The potential energy parameter C with the help of the wave function can directly be obtained from the expectation value of the energy operator for a given α so that

$$C = \frac{1}{2} \left(\sum_i \frac{|\langle i | Q | 0 \rangle|^2}{\epsilon_i - \epsilon_0} \right)^{-1} K \quad \dots(3.99)$$

A similar expression for the parameter B the mass associated with the collective vibration may be derived by studying the time dependent wave function $\phi(\alpha, t)$ corresponding to the value given by (3.96) which is being not explicitly dependent on time. Alternatively B may also be calculated by employing the problem of the collective mass with that of moment of inertia already discussed in (3.16). The angular momentum component J_x along the X -axis with the specified value J_x and $\dot{\phi}_x = \omega$ are conjugate variables. In the case of vibrational consideration $\dot{\alpha}$ plays the role of ω while the generalized momentum variable conjugate to α , quantum mechanically amounting to $-i\hbar \partial / \partial \alpha$ replaces J_x so that

$$\begin{aligned} B &= 2 \hbar^2 \sum_i \frac{\left| \langle i | \frac{\partial}{\partial \alpha} | 0 \rangle \right|^2}{\epsilon_i - \epsilon_0} \\ &= \frac{1}{2} \hbar^2 \left(\sum_i \frac{|\langle i | Q | 0 \rangle|^2}{\epsilon_i - \epsilon_0} \right)^{-2} \\ &\quad \sum_i \frac{|\langle i | Q | 0 \rangle|^2}{(\epsilon_i - \epsilon_0)^3} \quad \dots(3.100) \end{aligned}$$

The angular momentum component J_x along the x -axis having the specified value I_x and $\dot{\phi}_x = \omega$ are conjugate variables. In the case of vibrational problem $\dot{\alpha}$ plays the role of ω while the generalized momentum variable conjugate to α quantum mechanically amounting to $-i\hbar \partial / \partial \alpha$ replaces J_x . Then employing the relations given by (3.90), (3.99), (3.101) and (3.102), the energy $\hbar \omega$ implicitly may be expressed as

$$\begin{aligned}
2K = & \left(\sum_i \frac{|\langle i | Q | O \rangle|^2}{\epsilon_i - \epsilon_0} \right)^{-1} - \\
& - (\hbar \omega)^2 \left(\sum_i \frac{|\langle i | Q | O \rangle|^2}{\epsilon_i - \epsilon_0} \right)^{-2} \\
& \sum_i \frac{|\langle i | Q | O \rangle|^2}{(\epsilon_i - \epsilon_0)^3}
\end{aligned} \tag{3.101}$$

with the values of B and C it will be possible to calculate the collective transition probabilities B(E2). The E2 vibrations in single closed shell nuclei have been quite extensively calculated⁽⁵⁵⁾ and the computed values have been found to be in good agreement with the experimental results. Similarly calculations of γ -vibrations of the nuclei in the deformed region⁽⁵⁶⁾ have been carried out with a measure of success. Nevertheless the treatment outlined here involves various assumptions, firstly, the adoptability one expressed by (3.90), (3.92) and (3.93) of questionable validity, especially in the case of the vibrational state being in the very vicinity of regular two quasi-particle excitations. Moreover, the harmonicity approximation of (3.97) is one which is limited to a harmonic vibrational spectrum. Lastly, as the regular two quasi-particle states are not considered to be affected by the collective vibrational states, the number of redundant variables is increased thereby likely to increase vagueness in the picture. The vibrational states are in principle based on the regular two quasi-particle states.

In the method outlined above, some improvements have been brought about by introducing the random-phase approximation⁽⁵⁷⁾ which approximately takes into account the fact that the actual ground state in an even system due to the HQ_{40} terms may contain admixtures of four-, eight etc. quasi-particle states. Relative to this correlated actual ground state, denoted $|\tilde{O}\rangle$, excited states in general in which the vibrational state be found have components which correspond to the excitation of two quasi-particles as well as de-excitations of two quasi-particles. The equations of motion for amplitudes $\langle K | A^+_{\nu\nu'} | \tilde{O} \rangle$, are considered. The quantity $A^+_{\nu\nu'}$ is a

complex operator containing two quasi-particle creation operators. The term $|K\rangle$ signifies an excited state such as a vibrational one. The commutation rules of $A_{\nu\nu'}$ are expressed by

$$[A_{\nu\nu'}, A_{\nu_1\nu_1'}^\dagger] \simeq \delta_{\nu\nu_1} \delta_{\nu'\nu_1'} \quad \dots(3.102).$$

A numerical diagonalization procedure can give the eigenvalues $\hbar\omega$ and the corresponding eigen vector or the $A^\dagger$ components. With a further assumption that certain matrix elements of the separable interaction HQ could be ignored, or in other words, the exchange terms or those that spring from antisymmetrization of the wave function of the two interacting particles would lead to analytically simple equation for eigenvalue for $\hbar\omega$. In the case of degenerate j -shell the terms neglected are small being of the order $(2j+1)^{-\frac{1}{2}}$ and $(2j+1)^{-1}$ compared to the terms kept so that

$$2K \sum_i \frac{|\langle i | Q | 0 \rangle|^2 (\xi_i - \xi_0)}{(\xi_i - \xi_0)^2 - (\hbar\omega)^2} = 1 \quad \dots(3.103)$$

which in terms of quasi-particle formalism is interpreted to represent in case of an even-even nucleus an expression given by

$$2K F(\omega) \equiv 2K \sum_{\nu\nu'} \frac{|\langle \nu | q | \nu' \rangle|^2 (E_\nu + E_{\nu'})}{(E_\nu + E_{\nu'})^2 - (\hbar\omega)^2} (U_\nu V_{\nu'} - V_\nu U_{\nu'})^2 = 1 \quad \dots(3.104)$$

the Q being decomposed into its single particle components. $Q = \sum_i q_i$.

It may be pointed that an expansion in $\hbar\omega (\xi_1 - \xi_0)$ with a retention of terms upto second order enables an easy verification that (3.101) is obtained from (3.103).

From the solutions of (3.104) as exhibited in Figure (3.13) it can be found that $|K| \ll 1$ corresponds closely to the two quasi-particle energies $E_\nu + E_{\nu'}$. However, there may be solutions which are appreciably different with increasing $|K|$. In $T=0$ quadrupole vibrational states positive K is realised in which case the forces are attractive and

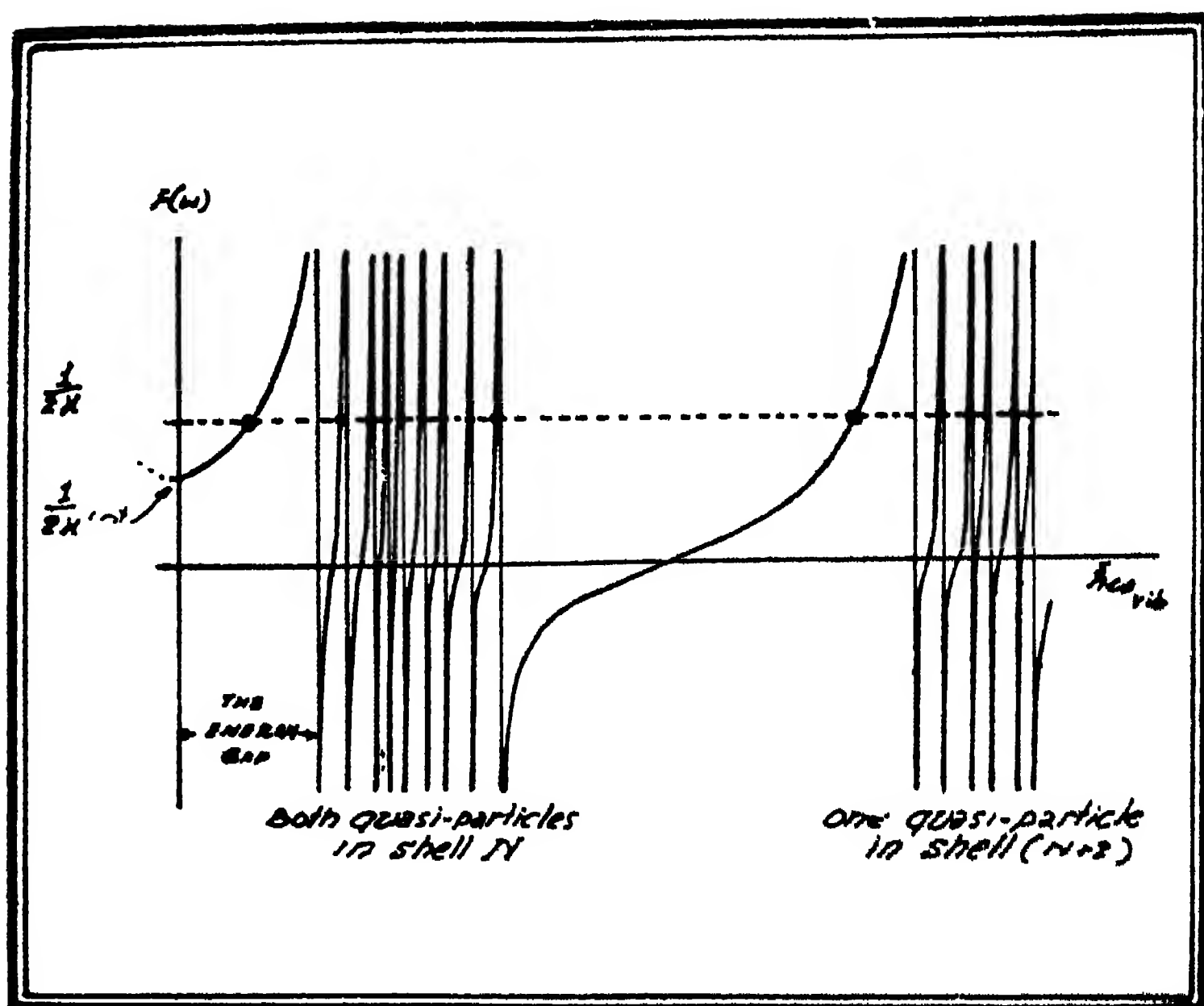


Figure 3.13

Schematic Figure of the Function $F(\omega)$ in Equation (3.104)

the frequencies decrease with increasing K . If K be larger than K_{crit} the lowest root becomes imaginary, implying thereby that the spherical configuration is by no means stable. If K be increasing and yet $K < K_{\text{crit}}$ two types of root collect a large fraction of the quadrupole oscillator strength for electromagnetic excitations from the ground state. One root may take place below the smallest sum $E_\nu + E_{\nu'}$, which corresponds to the low lying vibrational state in the spherical case and to the β —($I_3=0$) or γ —($I_3=2$) vibrational state in deformed nuclei. With an energy slightly below $2\hbar\omega_{\text{osc}}$, another type of collective E2 state is associated with excitations from the N shell to that of $N+2$. Negative case is realised if $T = |E|$ resonance and in this case the roots are pushed upwards and the collective strength is collected on one or a few roots slightly above the one shell transition energy⁽⁵⁹⁾.

In the case of spherical nuclei of Pb and Sn region, the difference between the frequencies obtained from equation (3.103) and those obtained by the adiabatic assumption from (3.101) is definitely small. This difference, however, is more pronounced in the deformed-actinide region especially when the vibrational states lie close to the regular two quasi-particle states. The change in vibrational frequency may be effected by employing (3.103) as compared to (3.101) by a readjustment of coupling constant in the latter equation the magnitude of which involves an uncertainty by at least the necessary amount in most cases. The random phase approximation has the principal advantage of being treated on the same footing in all cases whether it be purely intrinsic in character or collective or mixed.

The results of the random phase calculations with those obtained by a complete diagonalization of all two quasi-particle states may be compared following Tamm-Dancoff method which has earlier been applied⁽⁶⁰⁾ to the treatment of the Giant E1 state. This method though relatively successful in the prediction of the position of the collective E1 states existing at high excitation energy, is quite inadequate for the treatment of E2 and E3 collective states. The total oscillator strength computed within the limits of Tamm-Dancoff approximation is sometimes different by orders of magnitude from that obtained from the one resulting from energy-weighted sum rule, required to hold within any model which takes only nucleons into account of course disregarding meson effects as well as also for two particle interaction of non-exchange character. The violation of the sum-rule results from the negligence of the couplings to the higher quasi-particle states. The agreement with the sum rules is nevertheless very much improved in the case of random phase approximation of some ground state orientation. The energy weighted sum rule is found to hold exactly⁽⁶¹⁾ in the additional approximation implied in (3.104). On the basis of the approximations implied by this expression quadrupole vibrational modes can be calculated for spherical as well as for the deformed region. The regions of $\hbar \omega$ imaginary correspond to theoretical non-spherical equilibrium. This predicted instability in some case is upheld by the occurrence of a deformed region as in the

case for $150 < A < 190$. Such a non-spherical equilibrium shape is not found experimentally in other cases as near as $A=125$.

The collective E3 states have been dealt with in both Tamm-Dancoff method and in the random phase approximation^(62, 63). The methods by which the position and oscillator strength can be calculated are in general satisfactory. Even so the problem of the states characterized by one or more oscillator quanta largely remains unsolved. The random phase approximation becomes unwieldy technically to handle if directly adopted to treat states basically corresponding to four quasi-particle excitations relative to the ground state. With excitations of more quasi-particles, the validity of the random phase approximation becomes relatively more vague and uncertain⁽⁶⁴⁾.

A simple addition of another vibrational quantum as implied by the Hamiltonian (3.97), predicting a degenerate $I=0, 2, 4$ doublet in the two phonon quadrupole case does not in general correspond to the empirical situation. With the accumulation of experimental information on the problem of vibrational modes, the one of the higher vibrational modes is becoming an immediate pressing theoretical problem requiring on urgent solution.

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Phonon Coupling Models

4.1 Introduction

It is well-known that single particle models are applicable for spherical nuclei while the collective models are applicable for deformed nuclei. It is hard to really classify the nuclei rigidly into spherical and deformed types a considerable overlap being encountered especially at higher excitations. Classification based on the energy levels alone is sometimes difficult. Thus for odd mass spherical nuclei while the energy levels may indicate the applicability of a single particle model considerable discrepancies may be encountered on a detailed comparison of the static and dynamic electromagnetic moments. Several such cases have come to light in recent years and phenomenological models are developed to explain the level structures. Although several such models are developed, two of them need a special study. The model proposed by De Shalit⁽¹⁾, known as the core excitation model may be considered for odd mass spherical nuclei in which the coupling between the last odd-nucleon and the even-even core may be considered weak. On the other hand the model developed by Bohr⁽²⁾ and Chowdhury⁽³⁾ considers the coupling to be neither weak nor strong. This model is referred to as the intermediate coupling model of the unified type. In the following pages a brief outline of these models will be presented.

4.2 Core Excitation Model

A nucleus may be excited from its ground state by different ways of which the simplest may be described in the approximation of independent particle motion by the elevation of the

single particle from one state to another. These single particle excited states show up⁽⁴⁾ in reactions such as stripping, pick-up and possibly other reactions. An excitation which is complex is the one in which the ground state configuration remains unchanged. In this complex configuration the nucleons, however, change the relative orientation of their orbits. Such a situation may be observed in a case of $_{28}\text{V}^{51}$. The neutron number of this isotope being 28, which is a magic number, its ground state has $J=7/2$ belonging to the proton configuration $(1f_{7/2})^3$. The first excited state has $J=5/2$ which is believed to be of the same $1f_{7/2}$ configuration. Then consider the isotope $_{17}\text{Cl}^{38}$ with neutron number $N=21$. The ground state and the three lowest excited states are taken to be the four states of the configuration $(1d_{3/2}; 1f_{7/2})$ amounting to the configuration of one proton in $1d_{3/2}$ and one neutron in $1f_{7/2}$. A third class of excitations is that due to the collective motion of many nucleons⁽⁵⁾ which include collective rotations, vibration etc. which may combine in characteristic ways. In the regions of large deformations collective rotations generally represent the lowest excitation. In such cases it is possible to excite a single nucleon from one orbit in the deformed potential to another; the excited single particle state then forming the basis for a new rotational band.

Consider the ground state configuration of an odd-even nucleus described by $| (j_p)^2 J_p=0 j_n \rangle$ amounting to a description by a pair of protons in j_p coupled to $J_p=0$ and a neutron in j_p . Then the excitations described by $[(j_p)^2 J_p \neq 0 j]_J$ may be expected in addition to the single particle excitation which will be described by the configurations $| (j_p)^2 J_p \neq 0 j \rangle$. In such a state the neutron remains in its lowest state, the proton pair being decoupled and excited to a state, with $J_p \neq 0$ while J_p and j_n are then coupled to J , the total angular momentum.

As a step towards generalization of the mentioned mode of excitation, consider an even-even nucleus A with its ground $J=0$ followed by various excited states. Let a nucleon be added to this even-even nucleus. The ground state could be obtained if the odd nucleon is inserted in the lowest orbit of the average potential resulting in the core of A nucleons. Let the lowest single particle state in this average potential be supposed to be

high compared to the lowest excitation energy of the even-even core. It could then reasonably be assumed that the lowest excitations of the nucleus $A+1$ could be described by the odd-nucleon remaining in the lowest orbit and the core excited to the first excited state. A further clarification may be obtained by considering the even-even nucleus, in the average field produced by which, the lowest allowed proton orbit may be an $s_{1/2}$ orbit. It is actually found experimentally that $_{81}\text{Tl}^{203}$ has a $\frac{1}{2}^+$ ground state. Its level diagram (see Fig. 4.1) shows two excited states $3/2^+$ and $5/2^+$. These states are usually interpreted as those arising by promoting the proton (or rather a proton hole) from the single particle $s_{1/2}$ state to the single particle $d_{3/2}$ and $d_{5/2}$ states. The excitation energy of these states is comparable to that of the first excited state in Hg^{202} . If $3/2^+$ and $5/2^+$ in Tl^{203} were actually hole excitations it could be argued that there should separately exist the $3/2^+$ and $5/2^+$ states resulting from the coupling of the $s_{1/2}$ proton to the 2^+ state of Hg^{202} core. However, such states are not known around the excitation energy of 500 keV in Tl^{203} . Hence the two observed levels could indeed be resulting from coupling the $s_{1/2}$ proton to the 2^+ state of Hg^{202} core. This situation leaves ground for seeking the single hole excitations at higher energies.

A study of the nature of the excited states in Tl^{203} as well as those in other nuclei is further pursued to have a proper inkling of the actual situation. If these states were to be core-excitation coupled to the odd-nucleons in the lowest state the measured data on these levels could enable to make a deduction for further elucidation of the properties of the core in which case the picture of the spacings between single particle levels in these nuclei requires a modification. The empirical study of this problem has been tackled by De-Shalit¹. As a prerequisite for an analysis and interpretation of such data is made as indicative of the core excitations in odd-even nuclei. The information that can be drawn on the core-states from the available data on the excited states from some odd-even nuclei may be called, studied and discussed in order to enrich and widen the knowledge of the core-excitation in non-deformed odd-A nuclei.

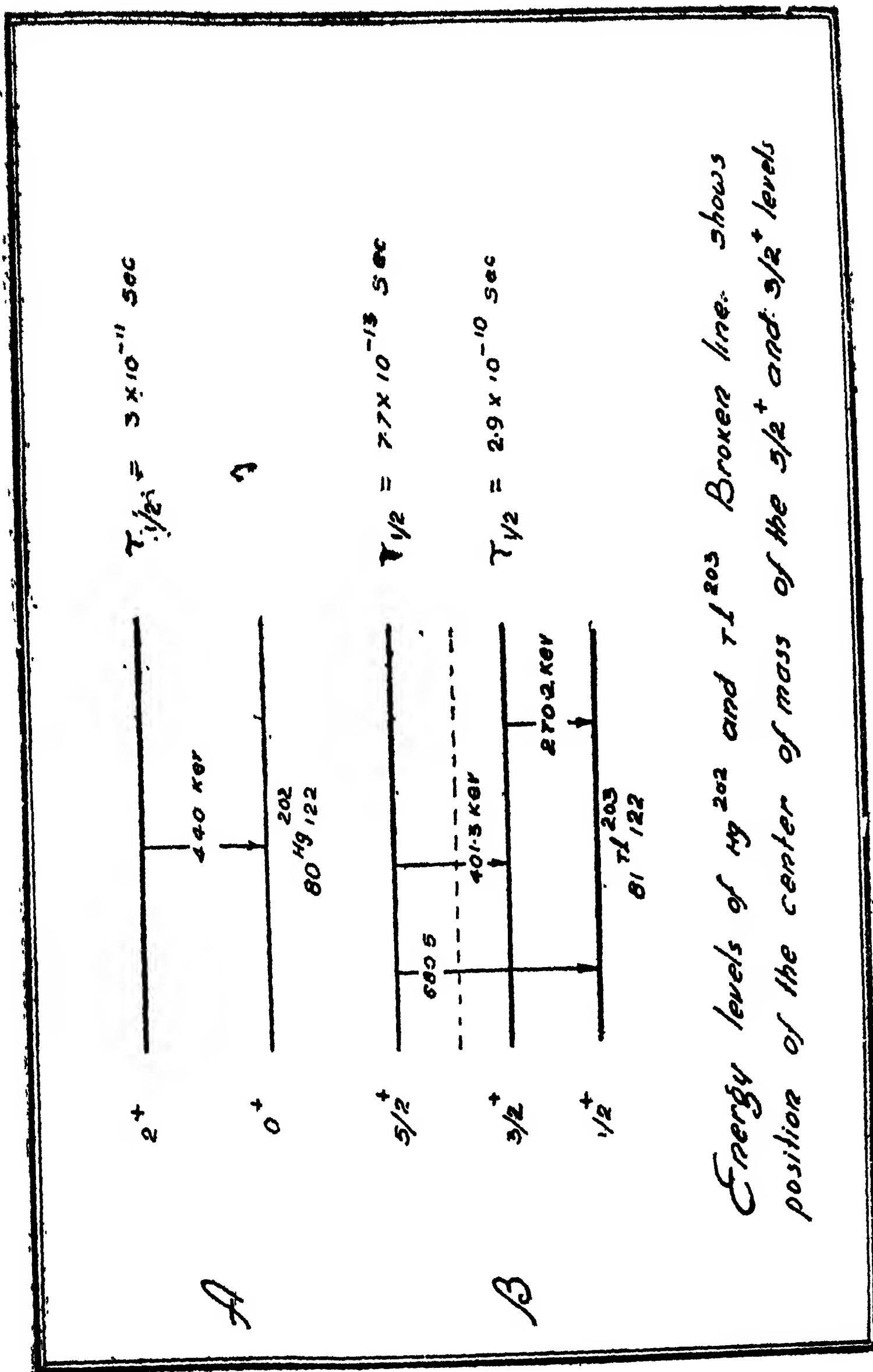


Figure 4.1

An investigation on the core-excitation in odd-mass nuclei may be carried out by describing the zeroth order wave functions of these nuclei by the ket vector $|J_c j, JM\rangle$, where J_c denotes the core angular momentum, j that of the odd particle and J the total angular momentum with $J_z = M$. In the absence of interaction between the core and the odd-particle, all the states characterized by the same pair of values J_c and j but different values of J (and M) are degenerate. These levels are referred as a core multiplet. In the case of particle core interaction, the degeneracy is removed within core-multiplet leaving then only the M -degeneracy.

The core state J_c in an odd-mass nucleus tends to be identified with the corresponding core-state J_c in the neighbouring even-even nucleus which could actually be done only with some reservation. The ground state $J_c = 0$ of an even-even nucleus, on expanding in terms of the single particle states of the self-consistent average field generally includes nucleons also in the state j . A nucleon added to the J orbit in accordance to the Pauli principle is rendered less available for the particles of the core. Hence the core state $J_c = 0$ in an odd-mass nucleus generally stands for something different from the core state $J_c = 0$ of the neighbouring even-even nucleus. The addition of a single particle to the j orbit however will not be decisive even if the core states represent a thorough mixture of the self-consistent single particle states. This situation is assumed to be the case and the antisymmetrization of the wave function $|J_c j, JM\rangle$ with respect to the exchange of the odd-particle and the particles of the core is treated negligible. This approximation may be of questionable validity.

The interaction between the odd-particle and core is a scalar and may be taken as the product of two tensors of degree k , $T^{(k)}(c)$ operating on the core degrees of freedom and $T^{(k)}(p)$ operating on the degrees of freedom of the particle. The general interaction may be looked upon as the sum of these products over all values of k being left quite general. Hence only one value of k at a time may obviously be taken into consideration.

The energy shift $\Delta E_k(J)$ of the state $|J_c j, J\rangle$ from its unperturbed Zero-th order energy to first order approximation

is given as

$$\begin{aligned}\Delta E_k(J) &= \langle J_c j J | T^{(k)}(c) \cdot T^{(k)}(p) | J_c j J \rangle \\ &= (-1)^{J_c + j + J} \langle J_c || T^{(k)}(c) || J_c \rangle \\ &\quad \times \langle j || T^{(k)}(p) || j \rangle \left\{ \begin{matrix} J_c & j & J \\ j & J_c & k \end{matrix} \right\}\end{aligned}$$

where

$$\left\{ \begin{matrix} j_1 & j_2 & j_3 \\ l_1 & l_2 & l_3 \end{matrix} \right\} = (-1)^{j_1 + j_2 + l_1 + l_2} W(j_1 j_2 l_2 l_1, j_3 l_3) \quad \dots(4.1)$$

The dependence of $\Delta E(J)$ noted as coming through universal functions so that from the orthogonality relations of Racah coefficients⁽⁶⁾ any interaction follows

$$\sum_k (2J+1) \Delta E_k(J) = 0 \quad \text{if } k \neq 0 \quad \dots(4.2)$$

Since the relation expressed by 4.2 does not involve monopole ($k=0$) the center of gravity of each multiplet coincides with its unperturbed position. In the case of monopole excitation ($k=0$), $E_0(J)$ is independent of J so that an interaction of this type does not produce splitting in the multiplet, consequently only resulting in shifting the multiplet as a whole by an energy:

$$\Delta E_0 = \frac{\langle J_c || T^0(c) || J_c \rangle}{(2J_c + 1)^{1/2}} \cdot \frac{\langle J || T^0(p) || j \rangle}{(2j + 1)^{1/2}} \quad \dots(4.3)$$

The quotient of the first term of the product in this expression (4.3) generally depends on the value of J_c . There are, however, two important cases for which the first term of the product involving the mentioned quotient is independent of J_c . If the core state consists of particles only in one orbit amounting to the core states being $| j_c^n J_c \rangle$, and if $T^0(C)$ is a sum of particle operators, that it can be shown that $\langle J_c || T^0(C) || J_c \rangle \approx (2J_c + 1)^{1/2}$. Similarly if a sequence of values of J_c arising from a collective motion be considered superimposed on the same intrinsic structure, then also $\langle J_c || T^0(C) || J_c \rangle \approx (2J_c + 1)^{1/2}$. In these cases, the separation between the center of mass of multiplets constructed on different core states J_c with the same single particle state j , should be equal to the separation between the unperturbed multiplets. Moreover, if the core states in the

odd-mass nucleus are identical with those of the neighbouring even-even nucleus, the separation of the center of mass of the multiplets ought to be identical with the separation between the corresponding states in the even-even nucleus. This rule is known as "center of gravity" theorem enunciated by Lawson and Uretsky(7). There is in general no rational basis to expect that this theorem holds precisely. In realistic cases, the value of $\langle J_c \parallel T^0(C) \parallel J_c \rangle / (2J_c + 1)^{\frac{1}{2}}$ is likely to be dependent at least to some extent on J_c . Also even if the core particle interaction contains no monopole part, the core-excitation of the odd-mass nucleus generally requires an energy other than the corresponding excitation in the even-even nucleus. However, the center of mass theorem can be expected to hold qualitatively as can be apparent when the empirical data is surveyed and discussed.

It could especially be simple when the odd-particle is in a state with $j = \frac{1}{2}$. J_c can be assumed to be of values 0, 2, 4 etc. with $j = \frac{1}{2}$. Hence each value of J determines a unique value of J_c . Moreover, for $j = \frac{1}{2}$, only $k=0$ and $k=1$ can contribute to the value of $\Delta E_k(J)$. As already mentioned, $k=0$ does not cause any splitting. Consequently the entire multiplet should be the outcome of the term with $k=1$ in the particle core-interaction. The existence of splitting and its magnitude are indicative of the measure of dipole interaction between the odd-particle and even-even core. Also if the odd-particle be in $s_{\frac{1}{2}}$ orbit then the only operator $T_{(p)}^k$ of velocity independent type for which $\langle s_{\frac{1}{2}} \parallel T(k) \parallel s_{\frac{1}{2}} \rangle$ does not vanish is $f(r_p) \vec{\sigma}_p$. Hence the multiplet splitting in this case is likely to measure the interaction of the spin of the odd-particle with the core.

It may be pointed that apart from the considerations on energies, the proposed mode of excitations also effect very strongly the electromagnetic transition probabilities between the different states. A detailed consideration necessitates the introduction of the notation for convenience. The $\Omega_k^{(k)}$ is the component of the tensor operator of degree k . The expectation value of the term $\Omega^{(k)}$ defines conveniently the

static multipole moment of order $k = 1, 2$ for which the $\Omega_o^{(1)}$ and $\Omega_o^{(2)}$ are given by

$$\begin{aligned}\Omega_o^{(1)} &= \sum_i g_{li} \vec{li} + g_{si} \vec{si} \\ \Omega_o^{(2)} &= (16\pi/5)^{\frac{1}{2}} \sum_i e_i r_i^2 Y_{20}(\theta_i, \phi_i)\end{aligned}\quad \dots(4.4)$$

where g_l and g_s are g-factors, e_i is the charge. The rate of emission of radiation of the corresponding multipolarity can be written in terms of the operators $\Omega^{(k)}$ as

$$T_{i \rightarrow f} = E^{2k+1} \mathcal{L}_k [I / (2J_i + 1)] | \langle f | \Omega^{(k)} | i \rangle |^2 \dots(4.5)$$

where i and f are the initial and final states respectively, J_i is the total angular momentum of the initial state, E the energy of the transition under consideration and $\mathcal{L}_k$ is a numerical constant. If E be given in MeV, the numerical values $\mathcal{L}_k$ turn out to be

$$\mathcal{L}_1 = 4.2 \times 10^{12} \text{ Sec}^{-1} \text{ MeV}^{-3} \quad \text{for M1 radiation} \quad \dots(4.6)$$

$$\mathcal{L}_2 = 1.23 \times 10^{13} \text{ Sec}^{-1} \text{ MeV}^{-5} \quad \text{for E2 radiation}$$

Each multipole operator $\Omega^{(k)}$ may be separated into a part due to the core and a part due odd-particle as

$$\Omega^{(k)} = \Omega_o^{(k)} + \Omega_p^{(k)}$$

so that the general matrix element of interest in the present context is obtained as

$$\begin{aligned}& \langle J_c' j, J_f | \Omega_o^{(k)} + \Omega_p^{(k)} | J_c j, J_i \rangle \\&= (-1)^{J_c' + j + J_f} \left[(2J_f + 1)(2J_i + 1) \right]^{\frac{1}{2}} \\& \times [\langle J_c' | \Omega_o^{(k)} | J_c \rangle \begin{Bmatrix} J_c' & J_f & j \\ J_i & J_c & k \end{Bmatrix} \dots(4.7) \\& + (-1)^{J_i - J_f} \langle j | \Omega_p^{(k)} | j \rangle \begin{Bmatrix} J & J_f & J_c \\ J_i & J & k \end{Bmatrix} \delta_{J_c J_c'}]\end{aligned}$$

Some special cases of equation (4.8) may be considered in detail by examining the magnetic moments and M1 Transition

Probabilities. In the case of transitions between states belonging to the same multiplet the quantity

$\frac{T_{i \rightarrow f}(M1)}{E^3}$, following De-Shalit¹ may be written as

$$\frac{T_{i \rightarrow f}(M1)}{E^3} = \mathcal{L}_1 (2J_f + 1) \left[\langle J_c \| \Omega_c^{(1)} \| J_c \rangle \begin{Bmatrix} J_c & I_f & j \\ J_i & J_c & I \end{Bmatrix} \right. \\ \left. + (-1)^{J_i - J_f} \langle j \| \Omega_p^{(1)} \| j \rangle \begin{Bmatrix} j & J_f & J_c \\ J_i & j & 1 \end{Bmatrix} \right] \dots (4.8)$$

This relation can be simplified by choosing $\Omega_p^{(1)} = J_c$ and $\Omega_c^{(1)} = j$. In the case of $J_f \neq J_i$, the left hand side of the expression 4.7 becomes zero. Setting down that $\langle j \| \vec{j} \| j \rangle = [j(j+1)(2j+1)]^{\frac{1}{2}}$, gives

$$[J_c(J_c+1)(2J_c+1)]^{\frac{1}{2}} \begin{Bmatrix} J_c & J_f & j \\ J_i & J_c & 1 \end{Bmatrix} \\ - [j(j+1)(2j+1)]^{\frac{1}{2}} \begin{Bmatrix} j & J_f & J_c \\ J_i & j & 1 \end{Bmatrix} \quad \text{for } J_f \neq J_i$$

The magnetic moment $\mu(j)$ of a system the angular momentum of which is j is given by

$$\mu(j) = j g = \left[\frac{2j}{(2j+1)(2j+2)} \right]^{\frac{1}{2}} \langle j \| \Omega^{(1)} \| j \rangle$$

Hence equation 4.8 can be written as

$$\frac{T_{i \rightarrow f}}{E^3} = \mathcal{L} (2J_f + 1) j(j+1)(2j+1) \\ \times \begin{Bmatrix} J_i & J_f & 1 \\ j & j & J_c \end{Bmatrix}^2 (g_c - g_p)^2 \dots (4.9)$$

As in the case of M1 transitions within a rotational band in deformed nuclei the ratio is independent of the specific values of g_c and g_p . If $g_c = g_p$, such transition probabilities vanish inasmuch as under this stipulation $\Omega^{(1)}$ becomes proportional to $\vec{J} = \vec{J}_c + \vec{j}$ so that the different Eigen states of J_2 cannot be connected.

It is required to know at least one static moment in order to obtain separate values of g_c and g_p . If the ground state of the odd-mass nucleus be assumed to be the state with $J_c=0$, then its g -factor is identical with g_p . Alternatively a measured factor of an excited state is capable of giving another combination of g_c and g_p .

From the relation given by 4.7 it is evident that if the ground state in an odd-mass nucleus has $J'_c=0$, the M1 radiation from the multiplet constructed on $J_c=2$ to the ground state is absolutely forbidden. However, great care and discretion should be exercised in checking this prediction. It may be possible that the description of nuclear state as $|J_c j, J\rangle$ is not complete. As an instance it may be stated that the ground state is expected to be $|0 j, J=j\rangle$ at least with a small admixture of $|2 j, J=j\rangle$ if $j \geq 3/2$. Also the state $|2 j, J\rangle$ can likewise have small admixtures of the state $|0 j', J\rangle$ where $j'=J$, etc. Even a small admixture of 2–3% in some cases can lead to an appreciable M1 rate for a transition which should have been forbidden.

Likewise it can be shown for E2 transitions from the multiplet $|J_c=2, j; J\rangle$ to the ground state $|J'_c=0 j; J_f=j\rangle$, that the following relation holds:

$$(T_i \rightarrow_f / E^5) = \left(\frac{a_2}{5} \right) | \langle 0 || \Omega^{(2)}_c || 2 \lambda |^2 \quad \dots (4.10)$$

This relation suggests that $T_i \rightarrow_f / E^5$ should be the same for all E2 transitions proceeding from the various members of a multiplet to the ground state. This value, moreover, should be equal to $T_i \rightarrow_f / E^5$ for $2^+ \rightarrow 0^+$ transitions in neighbouring even-even nuclei provided the core-states are identical with the corresponding states in the even-even nuclei.

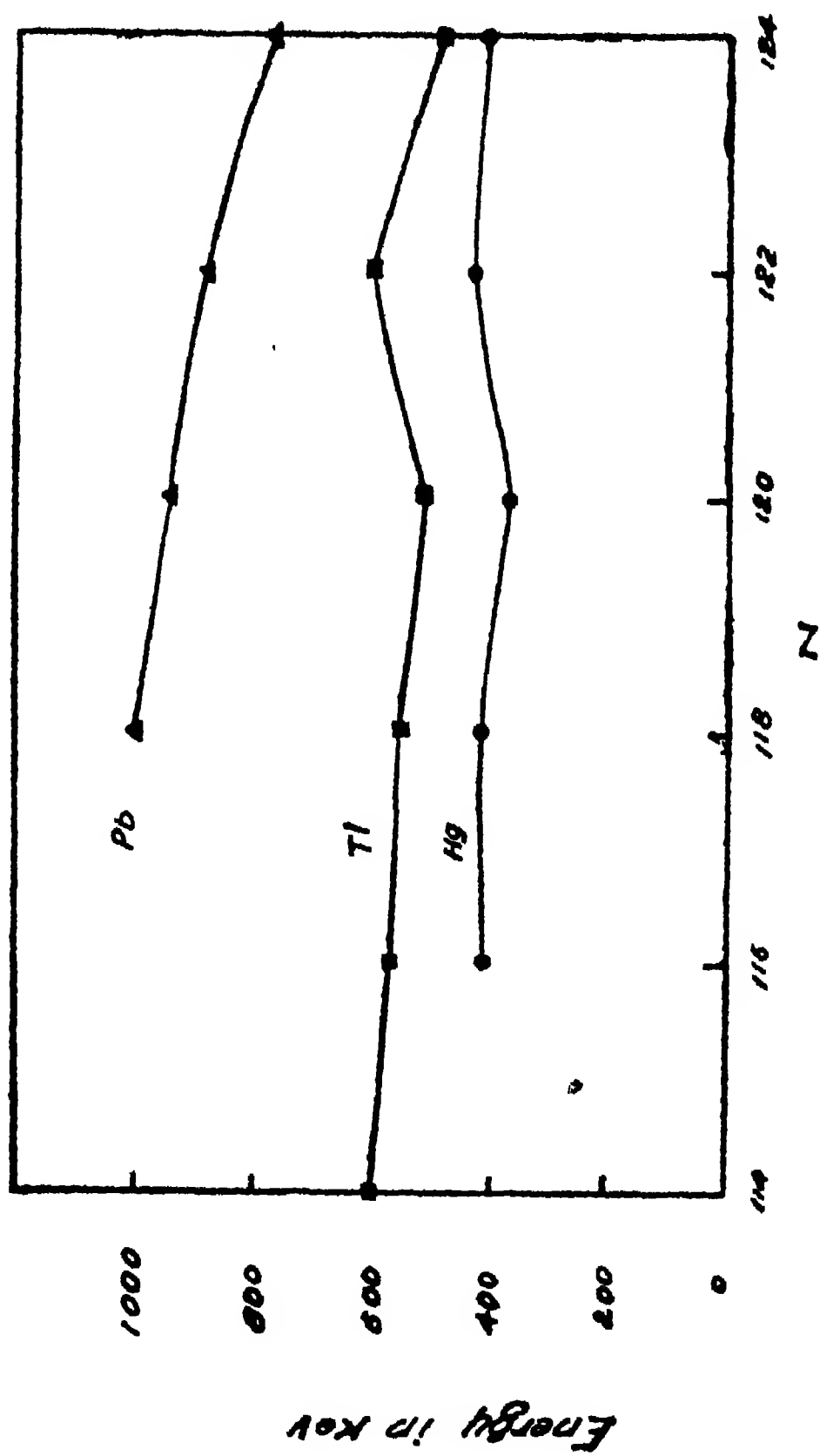
4.3 Analytical Studies of Some Experimental Observations

It is generally quite difficult to identify a group of levels in an odd-mass nucleus as belonging to one multiplet. The data on excited states are usually obtained by such methods involving the observations of levels with spins which do not differ

considerably from that of the ground state. Hence the multiplets with many components may not be realised in full thereby rendering the study difficult. There are, however, simple multiplets, the simplest of which are those arising from $j=\frac{1}{2}$. From the preceding discussion it can be found that in nuclei the ground state of which has $J=j=\frac{1}{2}$, there will be excited states with spins $J=3/2$ and $J=5/2$ with the parity identical to that of the ground state. All these doublets should be centered roughly around the energy of the first excited state in a neighbouring even-even nucleus and the electro-magnetic radiations should have peculiar features already mentioned in the previous section.

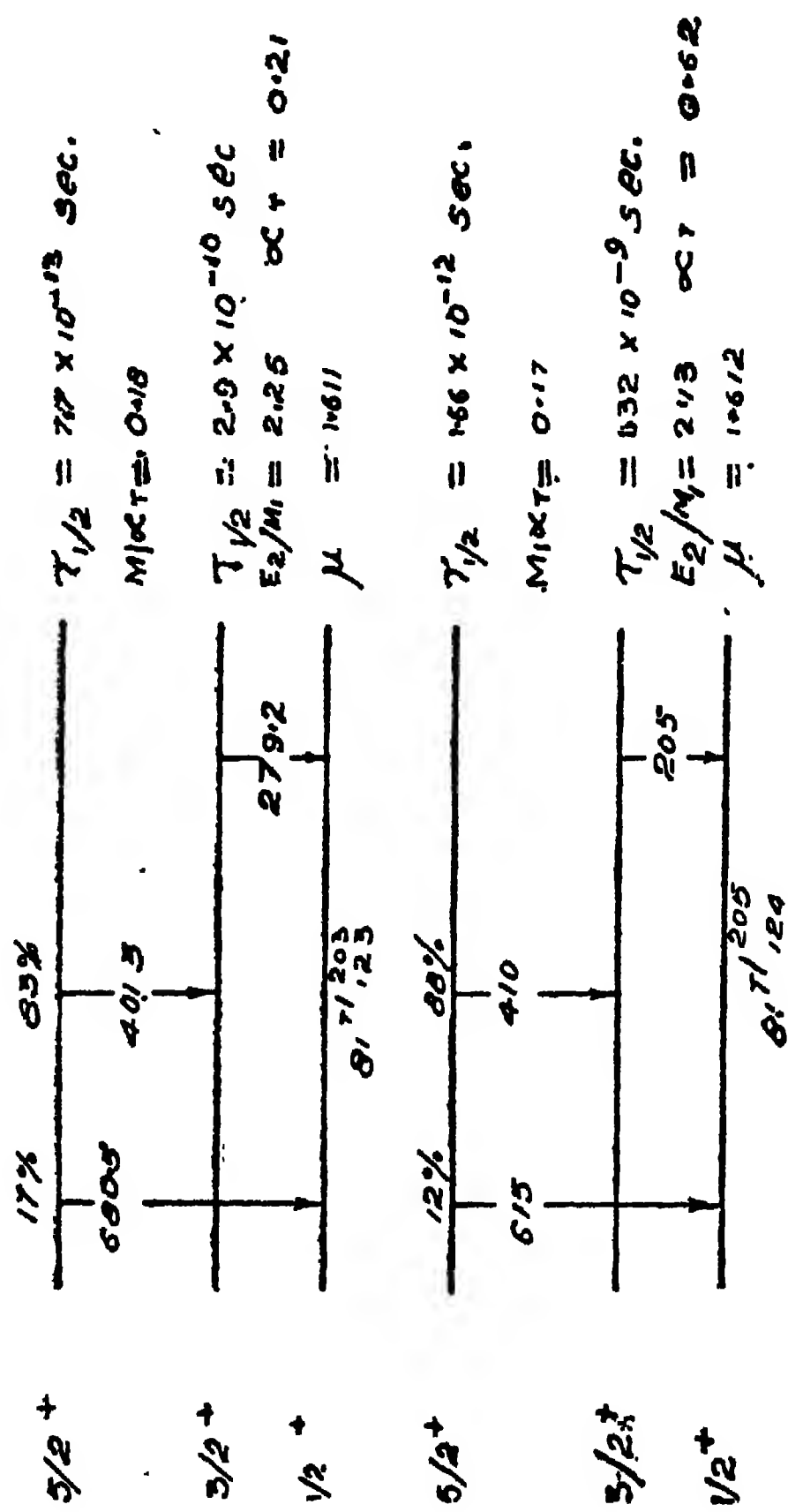
In the light elements the isotopic spin may bring in more complications. Disregarding such, in the isotopes with atomic number Z or neutron number N ranging from 39–49, the $p_{1/2}$ level shows up as the ground state. The isotopes, Se^{77} , Rh^{103} , Ag^{107} , Ag^{109} etc, are nuclei with large data to be analysed. There is yet another region with N ranging from 63–73, the odd-neutron number is in $s_{1/2}$ orbit, with Cd^{111} and Cd^{113} being chosen for detailed study. Then Tl isotopes for which the proton is in $s_{1/2}$ orbit and some odd-isotopes of Pt, Hg and also Pb for which the neutron is in $p_{1/2}$ orbit may be taken up for study.

To begin with let all the odd- A isotopes from Tl^{195} to Tl^{205} be taken into consideration. They show the same characteristic spectrum: $\frac{1}{2}^+$ as the ground state with $3/2^+$ and $5/2^+$, the lowest excited states. The center of gravity of $3/2^+ - 5/2^+$ doublets of these odd- A Tl isotopes as a function of neutron number is plotted and the plot is shown in Fig. 4.2. Therein the energies of the 2^+ states in Hg and Pb isotopes with equal number of neutrons are also shown. The trend of the Tl plot closely follows that of Hg suggesting thereby that the $3/2^+$ and $5/2^+$ excited states could be interpreted as a doublet constructed from the coupling of the $S_{1/2}$ proton to the 2^+ excited state of the core. The available measured data of the half lives and branching ratios of the Tl^{203} and Tl^{205} are indicated in Fig. 4.3. From these experimental data, the reduced E2 matrix elements $T_{i \rightarrow f}(E2)/E^5$ are deduced and tabulated as given in Table 4.1. The reduced matrix elements for the two



The 2⁺ excitation energies in the even-even isotopes of Hg and Pb and the position of the 3/2⁺ - 3/2⁺ center of mass of the odd a isotopes of Tl all as a function of the neutron number N

Figure 4.2



Energy levels of τ_{1203} and τ_{1205} showing the data used in the calculations. The branching ratios refer to total transitions.

Figure 4.3

E2 transitions in each of these Tl isotopes are equal to each other as predicted by the expression 4.10. The accuracy of the experimental data is only $\sim 15\%$.

The $3/2 \rightarrow 1/2$ M1 transition probability is very small, the reduced matrix elements being

$$T_{i \rightarrow f}(\text{M1})/E^3 = 3 \times 10^{10} \text{ sec}^{-1} \text{ MeV}^{-3} \text{ in Tl}^{203}$$

and

$$1.2 \times 10^{10} \text{ sec}^{-1} \text{ MeV}^{-3} \text{ in Tl}^{205}$$

In the case of single particle transition the expected corresponding value⁽⁸⁾ is

$$2.8 \times 10^{13} \text{ sec}^{-1} \text{ MeV}^{-3}$$

The highly reduced M1 rate is often associated with the possible /forbiddenness of the transition. If the $3/2^+$ state is interpreted as a $d_{3/2}$ state and the magnetic moment operator is taken as the conventional expression $\sum_i (g_{li} \vec{l}_i + g_{si} \vec{s}_i)$ then the

$d_{3/2} \rightarrow s_{1/2}$ transition probability vanishes. However, there might be corrections to the magnetic moment operator arising from exchange currents and the spin orbit interactions which could account for finite transition probabilities⁽⁹⁾. On the single particle picture, the large reduction of the $3/2 \rightarrow 1/2$ M1 matrix element in Tl isotopes is somewhat unexpected. The situation, however, is different with the identification of the $3/2^+$ state as $|2 \ 1/2 \ 3/2\rangle$. Now the M1 radiation is forbidden not on account of the special structure of the M1 radiation operator but on account of the operator being a vector (i.e. an irreducible tensor of degree 1) so that the states with $J_c = 0$ and $J_c = 2$ cannot be connected. Thus the M1 radiation operator has a tensorial character and is independent of the corrections to it. The existence of a slow M1 on the scene is indicative of the "impurity" of the state.

The $|3/2\rangle = A |2 \ 1/2 \ 3/2\rangle + (1-A^2)^{1/2} |0 \ 3/2 \ 3/2\rangle$ and the small component $|0 \ 3/2 \ 3/2\rangle$ could then lend to an M1 transition to ground state via the corrections to the magnetic operator.

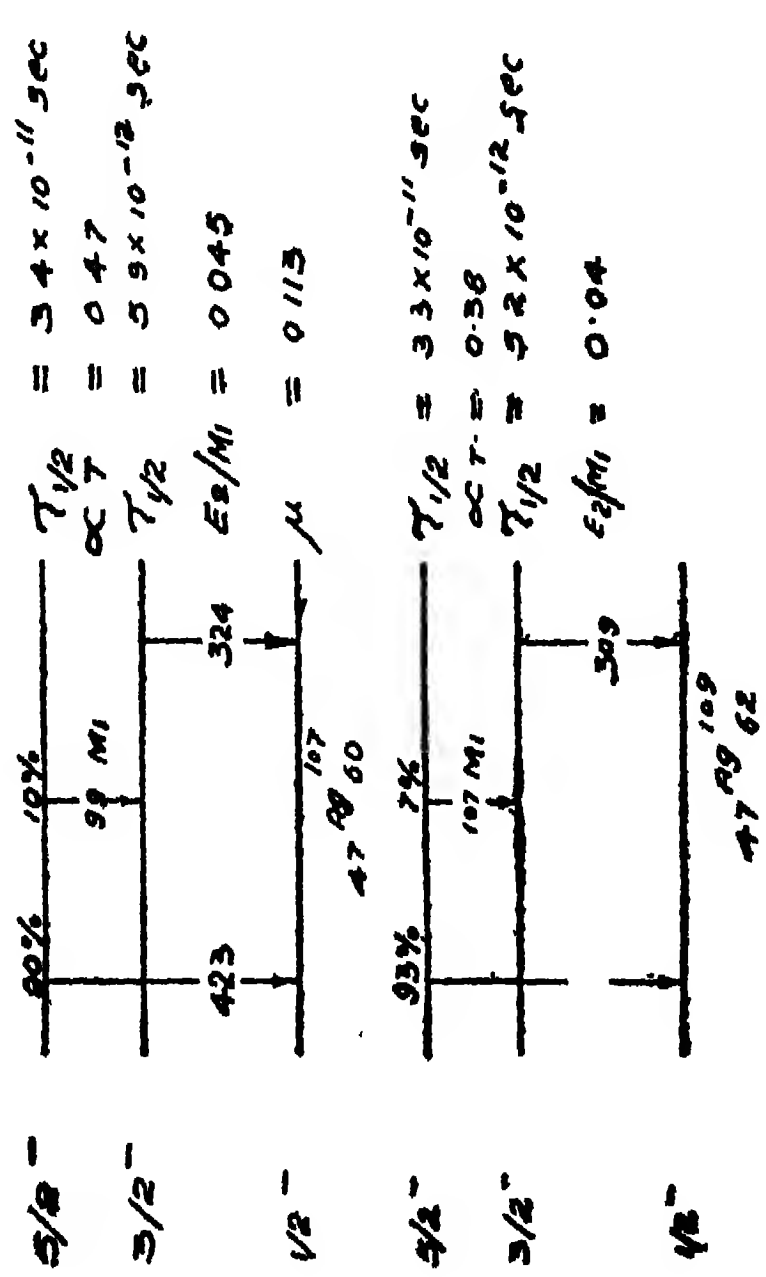
In the case of $5/2 \rightarrow 3/2$ M1 transition, the reduced matrix elements are $T_{i \rightarrow f}(\text{M1}) / E^3 = 1.2 \times 10^{13} \text{ sec}^{-1} \text{ MeV}^{-3}$ for Tl^{203} and $5.5 \times 10^{12} \text{ sec}^{-1} \text{ MeV}^{-3}$ for Tl^{205} . The errors in these especially for Tl^{205} are rather large inasmuch as they are derived in an indirect way by measuring the Coulomb excitation cross-section for the $5/2$ state and deducing the partial lifetime for the $5/2 \rightarrow 1/2$ and $5/2 \rightarrow 3/2$. The ratio being quite small for Tl^{205} leads to uncertainties in the interpretation of the numbers derived from it. A consideration of only Tl^{203} with the employment of the numerical values of $\mathcal{L}_1$ for M1 radiation and $\mathcal{L}_2$ for E_2 radiation given by 4.6 yields $(g_c - g_p) \approx \pm 2.6$. The g_p values derived from observed ground state magnetic moment give the value of g_c as

$$g_c = 0.6 \text{ nm}$$

The estimation of the limits of error on this small number is rather difficult. However, a value of $g_c = Z/A = 0.4$ is in fact cannot be excluded by the experimental results on Tl^{203} . The analysis of Tl^{205} on similar lines yields 1.6. As already pointed out the uncertainties in this case are too large to allow any conclusion.

The single particle assignments for the levels $1/2^+$, $3/2^+$, $5/2^+$ in the case of Tl isotopes would also make the $3/2^+ \rightarrow 1/2^+$ M1 transition a slow one (l -forbidden) whereas the $5/2^+ \rightarrow 3/2^+$ a "normal" M1 transition $d_{5/2} \rightarrow d_{3/2}$. Hence some of the special features of the coupling in this case are observed. If the nuclei the ground state of which is $1/2^-$ are considered the situation is different. They are, as explained before, expected to have an excited "doublet" $3/2^-$ and $5/2^-$ so that the single particle interpretation now is $p_{1/2}$ for the ground state and $p_{3/2}$ and $f_{5/2}$ for excited states. On the basis of this interpretation, the $3/2^- \rightarrow 1/2^-$ is normal M1 transition whereas the $5/2^- \rightarrow 3/2^-$ should be the l — forbidden transition. If the assumption of a "core doublet" is adopted, the situation, however, is reversed so that the $3/2^- \rightarrow 1/2^-$ is core forbidden whereas the $5/2^- \rightarrow 3/2^-$ M1 should proceed with a rate proportional to $(g_c - g_p)^2$.

Such a case is likely to be found in the Ag isotopes, the concerned levels of which are illustrated in Fig. 4.4. The



Energy levels of Ag^{107} and Ag^{109} showing the data used in calculations. The branching ratios refer to total transitions.

Figure 4.4

doublet center of mass in Ag^{107} for the energies $E_1 = 324$ keV with spin $J_1 = 3/2$ and $E_2 = 423$ keV with spin $J_2 = 5/2$ is obtained from the weighted average of the two levels as 383 keV and likewise in Ag^{109} for the energies $E_1 = 309$ keV with spin $J_1 = 3/2$ and $E_2 = 416$ keV with spin $J_2 = 5/2$ is obtained from weighted average of the other two levels as 373 keV. The mentioned energies may be compared with 2^+ excitation of 513 keV in $_{46}\text{Pd}_{60}^{106}$, 630 keV in $_{48}\text{Cd}_{60}^{108}$, 453 keV in $_{46}\text{Pd}_{62}^{108}$ and 656 keV in $_{48}\text{Cd}_{62}^{110}$. The reduced E2 transition probabilities are given in Table 4.2. These values lie within the range of values obtained for the neighbouring even-even nuclei thereby confirming the relation given by equation 4.10.

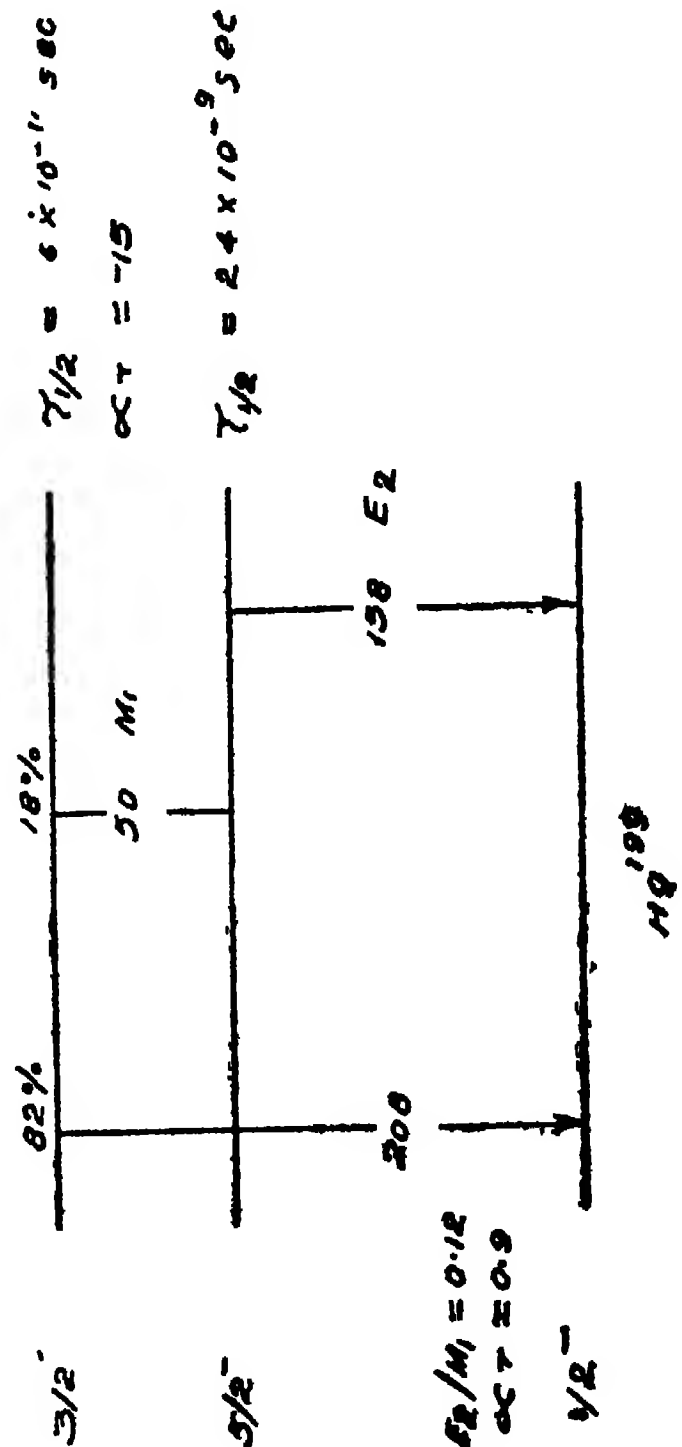
The $3/2^- \rightarrow 1/2^-$ transition should be forbidden and as a matter of fact the states are not expected to be pure and a slight admixture of the type $|0p_{1/2} \frac{3}{2} \rangle$ in $|2p_{1/2} \frac{3}{2} \rangle$ will cause an M1 radiation differing from the case of Tl the ground state of which is $\frac{1}{2}^+$ where the admixture could only contribute via the main part of the exchange current correction to the M1 radiation operator. Hence this effect could be expected to be considerably large. The observed matrix elements for $3/2^- \rightarrow 1/2^-$ transition are $T_{i \rightarrow f}(\text{M1})/E^3 = 3.3 \times 10^{12} \text{ sec}^{-1} \text{ MeV}^{-3}$ in Ag^{107} and $4.3 \times 10^{12} \text{ sec}^{-1} \text{ MeV}^{-3}$ in Ag^{109} . The single particle estimates⁽¹⁰⁾ in these two cases are $2.8 \times 10^{13} \text{ sec}^{-1} \text{ MeV}^{-3}$. The transitions are in this way slowed down considerably by an order of magnitude.

The $5/2^- \rightarrow 3/2^-$ M1 transition is / forbidden on the single particle assignment for these levels. They are, however, allowed if the two states are the components of a "core doublet". Then the reduced matrix elements could be obtained as $T_{i \rightarrow f}(\text{M1})/E^3 = 1.4 \times 10^{12} \text{ sec}^{-1} \text{ MeV}^{-3}$ for Ag^{107} and $0.9 \times 10^{12} \text{ sec}^{-1} \text{ MeV}^{-3}$ for Ag^{109} . If these are compared with the expected single particle rates for a $5/2 \rightarrow 3/2$ transition, $T_{i \rightarrow f}(\text{M1})/E^3 = 3.4 \times 10^{13} \text{ sec}^{-1} \text{ MeV}^{-3}$, they appear even more hindered than the $3/2^- \rightarrow 1/2^-$ transition previously discussed. In the case of Ag isotopes $(g_c - g_p)^2$ turns out to be very small. Consequently the

$5/2^- \rightarrow 3/2^-$ M1 transition is likely to reflect more the smallness of $(g_c - g_p)^2$ rather than the forbiddenness of a $f_{5/2} \rightarrow p_{3/2}$ transition. The employment of equation (4.9) gives $\pm 0.91 \pm 0.73$ for the values of $(g_c - g_p)$ for Ag^{107} and Ag^{109} respectively leading thereby to $g_c = 0.7$ nm and 0.5 nm for Ag^{107} and Ag^{109} respectively. From the similarity between these values for g_c and that obtained for Tl^{208} given by 0.6 nm, core-doublet interpretation is taken to have been further supported¹. Also the reduced matrix elements of $3/2^- \rightarrow 1/2^-$ and the $5/2^- \rightarrow 3/2^-$ radiations are seen as of the same order of magnitude. Therefore, the interpretation of the one as a normal M1 and the other as a 1 forbidden M1 is rather difficult. On the basis of the "core doublet" interpretation the smallness of the rate of the normal transition is attributed to the static moment of a $p_{1/2}$ proton whereas the latter to a near cancellation by the orbital moment.

The energy levels of Hg^{199} represented in Fig. 4.5 give the relevant data. It may be observed that the E2 reduced matrix elements for the $5/2^- \rightarrow 1/2^-$ and the $3/2^- \rightarrow 1/2^-$ are practically identical with each other and equal to those of Hg^{200} and Hg^{202} . It is interesting to note that the $3/2^- \rightarrow 1/2^-$ M1 radiation in Hg^{199} gives $T_{i \rightarrow f}(\text{M1})/E^3 = 4 \times 10^{+11} \text{ sec}^{-1} \text{ MeV}^{-3}$ compared with an expected single particle value of $2.0 \times 10^{18} \text{ sec}^{-1} \text{ MeV}^{-3}$ amounting to a hindrance by a factor 50. The $3/2^- \rightarrow 5/2^-$ M1 radiation, on the other hand gives $T_{i \rightarrow f}(\text{M1})/E^3 = 1.0 \times 10^{12} \text{ sec}^{-1} \text{ MeV}^{-3}$. This result with the observed magnetic moment of the ground state of Hg^{199} leads to a value of $g_c = 0.4$ nm which is in close agreement with other values obtained for g_c . These facts indicate the superiority of the "core doublet" interpretation over that of single particle excitation in odd-even nuclei.

It was suggested⁽¹⁾ that the first few excited states in Au^{197} would be described quite well by coupling the odd $d_{3/2}$ proton (hole) to an excited 2^+ core. The lowest available state for the 79th proton is probably a $d_{3/2}$ which may mean that the lowest excitation in Au^{197} involves the excitation of the core rather than the excitation of the single particle. The importance of



Energy levels of Hg^{199} showing the data used in the calculations. The branching ratios refer to total transitions

Figure 4.5

ascertaining core excitation has been already described⁽¹¹⁾. With the recent available data a reevaluation of the applicability of the core excitation model to Au^{197} has been made in order to ascertain the scope of its validity.

The figure 4.6 gives the levels together with the relevant experimental data. The core excitation attributes the following wave functions for the states under consideration

$$\begin{aligned}
 1 \text{ g.s.} &= |3/2>_1 = A |0 \ 3/2; 3/2> + \sqrt{1-A^2} |2 \ 3/2; 3/2> \\
 2: 77 \text{ keV} &|1/2> = |2 \ 3/2; 1/2> \\
 3: 269 \text{ keV} &|3/2>_2 = A |2 \ 3/2; 3/2> - \sqrt{1-A^2} |0 \ 3/2; 3/2> \\
 4: 278 \text{ keV} &|5/2> = |2 \ 3/2; 5/2> \quad \dots(4.11) \\
 5: 549 \text{ keV} &|7/2> = |2 \ 3/2; 7/2>
 \end{aligned}$$

Here, $|J_0, j, J>$ stands for a state in which a core in the state J_0 is coupled to the odd-particle in the state j to produce a total angular momentum J . In the extreme case $A=1$, however, a small deviation of A from unity as pointed⁽¹¹⁾ which indicates a coupling of the core states via their interaction with the odd-nucleon may be expected to exist. This picture will retain its intuitive meaning if $1-A^2 \ll 1$.

All the electro-magnetic properties involving the five states in (4.11) in terms of a few parameters can be calculated by employing the notations in references 1 and 11. Confining to magnetic dipole and electric quadrupoles and considering both static moments and transitions, these properties can be described in terms of the parameters:

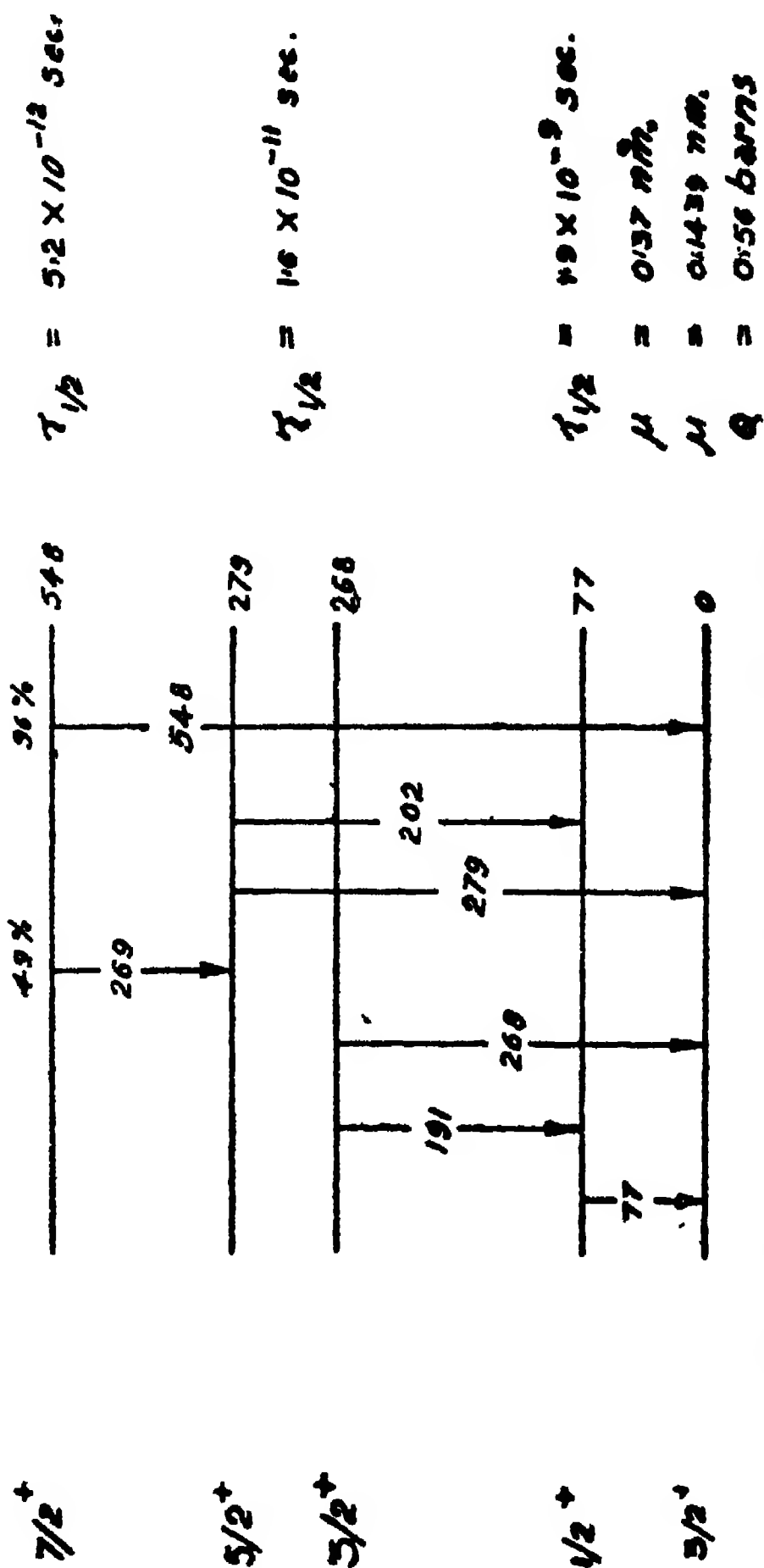
$$\begin{aligned}
 g_p = 0.083 & \quad \text{the Shimidt } g \text{ factor of the odd} \\
 & \quad \text{d3/2 protons} \quad \dots(4.12)
 \end{aligned}$$

$$g_0 = \quad \text{the } g \text{ factor of the } 2^+ \text{ state of the even-even core} \dots(4.13)$$

$$\begin{aligned}
 \Omega_p = <3/2||\Omega_p^{(2)}||3/2> & \quad \text{the reduced quadrupole matrix ele-} \\
 & \quad \text{ment for 4 odd-proton} \quad \dots(4.14)
 \end{aligned}$$

$$\begin{aligned}
 \Omega_{22} = <2||\Omega_0^{(2)}||2> & \quad \text{the same for the } 2^+ \text{ state of the core} \\
 & \quad \dots(4.15)
 \end{aligned}$$

$$\begin{aligned}
 \Omega_{20} = <0||\Omega_0^{(2)}||2> & \quad \text{the reduced quadrupole transition mat-} \\
 & \quad \text{rix element for the core} \quad \dots(4.16)
 \end{aligned}$$



Positive parity states in Au-197.

Figure 4.6

The four parameters, (4.13), (4.14), (4.15) and (4.16) together with A in (4.11) are expected to account the 5 static magnetic moments, 4 static quadrupole moments 6 M1 transitions and 8 E2 transitions experimentally derived. Since not all the data mentioned are available, the model can be put to partial test. Moreover, the assumption that the wave functions 4.11 describe the states considered with absolute precision is very naive. Hence an allowance is to be made for the presence of other reasonable admixtures with small amplitudes which in turn limits in the choice of the data on which the model can be tested⁽¹¹⁾.

Therefore, a procedure was adopted to determine Ω_p , Ω_{22} and Ω_{20} using the 548 keV E2 transition from $7/2^+$ state to the ground state, the 279 keV E2 transition from $5/2^+$ state to the ground state and the 202 keV E2 transition from $5/2^+$ state to the $1/2^+$ state. It may be mentioned that the 548 keV transition has no M1 admixture inasmuch as $\Delta J = 2$ and it decays predominantly through this channel so that no uncertainties of branching ratios come in. The 202 keV transition has been rather well-known as a pure E2.

The unique values Ω_p , Ω_{22} and Ω_{20} for any given value of A can be used to determine the partial E2 life for the 77 keV from the $1/2^+$ state to the ground state. With total lifetime of this state, the M1 partial lifetime and consequently δ^2 and $g_c - g_p$ may be computed from the relation given by

$$\frac{T_{i \rightarrow f}(\text{M1}, 1/2 \rightarrow 3/2)}{\omega^3 E_\gamma^3} = 3 (1-A)^2 (g_c - g_p)^2$$

This value is consistent with this model to take for g_p its unperturbed Schmidt value so that all the parameters are now fixed. Then the other quantities may be calculated and compared with the experimental data. The results obtained for three different values of A are given in Table 4.3. The errors quoted are those due to experiment on the lifetimes used as a starting point. The last column gives the experimental values of the calculated quantities. The quantity

$B(E2, (3/2)_2 \rightarrow (3/2)_1)$ given by

$$B(E2) = \frac{T_{\gamma}(E2) \text{ Sec}^{-1}}{[E_{\gamma}(\text{MeV})]^5 \times 1.23 \times 10^{13}} \quad \dots(4.17)$$

was calculated because of the quenching of the E2 in the $(3/2)_2 \rightarrow (3/2)_1$ transition. In the case of $A = 1$

$B(E2, (3/2)_2 \rightarrow (3/2)_1) = 0.22$ is obtained in the same units.

From table 4.1, it can be seen that $A^2 \approx 0.93$ gives a good fit between the calculated quantities and their experimental values with the possible exception of $\delta^2(191)$. Moreover, it is possible to deduce the quadrupole moment from the value of Ω_P as

$$Q(d_{3/2}) = (0.21 \pm 0.20) \text{ barns}$$

which should be compared with the single particle formula for

$$\begin{aligned} (d_{3/2})^3 Q &= \langle r \rangle^2 (2j - 1) / (2j + 2) \\ &= 0.21 \times 10^{-24} \text{ Cm}^2 \text{ if } r = 1.25 A^{1/3} \text{ fm} \end{aligned}$$

The quadrupole moment for the 2^+ state of the core turns out to be about (1.2 ± 0.2) barns which could be compared with the quadrupole moments of 2^+ states in the neighbouring even nuclei when they become available.

The nearness of A to unity justifies a comment judging from the width of the core multiplet which is of the order 500 keV and noting that it is comparable to the excitation energy of the 2^+ state, A^2 might be expected to be around 0.5. It was, however, noted⁽¹⁾ that the particle core interaction in this region of Au and Tl appears to be dominated by a dipole-dipole interaction. If it were so the smallness of $(1 - A^2)$ would be expected to follow. In the case of dipole-dipole force $1 - A^2 = 0$ to a first order in perturbation theory.

The excited states of Au^{197} as well as others were studied⁽¹²⁾ and the result led to a structure completely different from the one given by De-Shalit for some of the states considered. In

the former the $\frac{1}{2}^+$ state at 77 keV interpreted as single particle $s_{\frac{1}{2}}$ state has an exceptionally small value for $\mu(\frac{1}{2}^+)$ of 0.25–0.26 nm as compared with a Schmidt value of 2.79 nm for the pure $s_{\frac{1}{2}}$ single particle state. However, in the former, the ground state has been found to yield $\mu(3/2)=0.84–0.92$ nm as against $\mu_{\text{experimental}}(3/2)=0.144$. Further the calculated ground state quadrupole moment comes to 1.07 barns compared with the measured value 0.56 barn. On this basis it will be hard to explain the equality of the experimental reduced matrix elements for the E2 transitions from the $7/2^+$, $5/2^+$ and $\frac{1}{2}^+$ states to the ground state. The equality of these three reduced rates is an essential feature of the core excitation model. Consequently the core excitation model at least in the case of Au^{197} is said to offer a more consistent description of the electromagnetic properties of the nucleus. It has also enabled to successfully explaining the static moments in terms of transition probabilities. The values derived thereby for the bare single particle moments are in good agreement with the Schmidt value for the magnetic moment and with the unrenormalized quadrupole moment. The experimental error on the various quantities given in Table 4.3 are too large even in well-studied nucleus like Au^{197} . If lifetimes, conversion coefficient branching ratios and multipole mixing ratios be determined with improved accuracy by a factor 3–5, improved information may be obtained. The elucidation of the excited states is likely to contribute an improved determination of the actual position of the single particle states and to a deeper understanding of the nature of the collective 2^+ states in symmetrical even-even nucleus.

The proposal for a simple model for odd-A nuclei of the spherical type in which the particle is assumed to remain in its orbit, the core being excited has already been dealt with⁽¹⁾ in some detail by explaining the resulting characteristics and those of the transition probabilities. In the case of Au^{199} a finite amount of mixing in like-spin states has to be taken into consideration⁽¹¹⁾.

The intensities of the 269 keV and 202 keV transitions besides those of 77 keV and 191 keV transitions of Au^{197} have been

determined accurately employing improved techniques^(13, 14). The excited state ($\frac{1}{2}^+$) at 77 keV consists of 92% $s_{\frac{1}{2}}$ while the De-Shalit's theory does not allow the existence of any particle excitation in the $\frac{1}{2}^+$ state. A compromise between these two extreme points of view has been attempted by considering the wave functions of the low energy states as particle and particle $\frac{3}{2}^+$ plus phonon functions. The large B(E2) value for the 202 keV transition can be accounted by assuming an admixture of the particle $\frac{1}{2}^+$ plus phonon in the $\frac{5}{2}^+$ state. The wave functions then are expressed as

$$\begin{aligned}
 | \frac{3}{2} >_1 &= A | 0 \frac{3}{2} \frac{3}{2} > + (1-A^2)^{\frac{1}{2}} | 2 \frac{3}{2} \frac{3}{2} > \\
 | \frac{1}{2} > &= B | 0 \frac{1}{2} \frac{1}{2} > + (1-B^2)^{\frac{1}{2}} | 2 \frac{3}{2} \frac{1}{2} > \\
 | \frac{3}{2} >_2 &= A | 2 \frac{3}{2} \frac{3}{2} > - (1-A^2)^{\frac{1}{2}} | 0 \frac{3}{2} \frac{3}{2} > \\
 | \frac{5}{2} > &= C | 2 \frac{3}{2} \frac{5}{2} > + D | 0 \frac{5}{2} \frac{5}{2} > + (1-C^2-D^2)^{\frac{1}{2}} | 2 \frac{1}{2} \frac{5}{2} > \\
 | \frac{7}{2} > &= E | 2 \frac{3}{2} \frac{7}{2} > + (1-E^2)^{\frac{1}{2}} | 0 \frac{7}{2} \frac{7}{2} >
 \end{aligned}$$

From the values of B(M1) and B(E2) for the different transitions connecting the states and the magnetic moment of the ground state and solving the equations. Raju and Lakshminarayana⁽¹⁵⁾ derived the values for the constants as

$$\begin{array}{ll}
 A^2 = 0.97 & B^2 = 0.38 \\
 C^2 = 0.972 & D^2 = 0.015 \\
 E^2 = 0.76 & \\
 1-A^2 = 0.03 & 1-B^2 = 0.62 \\
 1-E^2 = 0.24 & 1-C^2-D^2 = 0.193
 \end{array}$$

It thus appears that pure core excitation approach may not be adequate and some mixing of particle excitations has to be considered.

4.4 Unified model in intermediate coupling

The core excitation model considers an odd-mass nucleus as being made up of an even-even core and one extra particle..

The particle is assigned to an angular momentum state based on shell model consideration and the core is assumed to be excited to produce low energy excited states of the nucleus. Since the low energy excitation of an even-even nucleus of the spherical type consist of surface oscillations, the resultant angular momentum are coupled to the particle angular momentum to yield the core excitation multiplet. The degeneracy of the core excitation multiplet is removed by considering a core particle interaction as outlined in the previous section. In formulating the core excitation model the unpaired odd-particle is assigned a definite angular momentum state. It is, however, established in direct reaction studies that although orbital assignment on the basis of shell model is possible as a general rule there exists a finite probability of any state being occupied or unoccupied. As such it is probably more meaningful to consider the possibility of assignment to the last odd-particle any angular momentum belonging to the major shell. In the core excitation model the coupling strength of the core and the particle was assumed to be small. However, the interaction strength particularly when a large number of angular momentum states are involved may not be weak. In nuclei away from the deformed regions it is also not logical to consider strong coupling. An intermediate coupling strength may therefore be more reasonable. Such unified model calculations in intermediate coupling were originally suggested by Bohr⁽²⁾ and the details were worked out by Chowdhury⁽³⁾. A number of papers were subsequently published⁽¹⁶⁻²⁵⁾ on the application of this model for various nuclei. A detailed description of the formalism was furnished by Heyde and Brussaard⁽¹⁷⁾ and is outlined briefly here.

The Hamiltonian of the system is assumed to be separable and is given by

$$H = H_c + H_p + H_{int}$$

where H_c and H_p denote the collective and the single particle parts which H_{int} represent the contribution resulting from the coupling of the core and the particle. The usual form of H_c generates phonon vibrations associated with the energy $\hbar\omega$, ω^2

being the ratio of the surface stiffness and the initial parameters C and B . The collective part produces equally spaced phonon states E_N given by

$$E_N = (N + \frac{5}{2}) \hbar \omega$$

The single particle Hamiltonian describes the motion of the extra core nucleon in an effective spherical potential and generates the usual single particle states in shell model description. In view of the uncertainty of the potential in size as well as the radial form factor of the effective potential, it is usual to treat the single particle energies as parameters in the calculation. Alternatively the single particle spacings can also be inferred from the experimental energy spectra in the region of interest. The interaction Hamiltonian is assumed to be given by

$$H_{\text{int}} = -k(r) \sum_{\mu} \langle \mu \rangle Y_2^{\mu}$$

The first factor $k(r)$ is related to the radial dependence of the potential and the radial matrix element is usually assumed to be in the range 20 to 40 MeV. In terms of the phonon creation and annihilation operators b and b^+ the interaction Hamiltonian may be written as

$$H_{\text{int}} = -\sqrt{\frac{\pi}{5}} \zeta \hbar \omega \sum_{\mu} \left(b_{\mu} + (-1)^{\mu} b_{-\mu}^+ \right) Y_2^{\mu}$$

where ζ is a dimensionless parameter given by

$$\zeta = k \left\{ \frac{5}{2\pi \hbar \omega C} \right\}^{\frac{1}{2}}$$

representing the interaction strength. A value in the region 1 to 4 is usually employed in the intermediate coupling calculations.

The coupling of the odd-particle represented by a wave function $|jm\rangle$ with the phonon vibration represented

by $|NR M_R\rangle$ is considered in a set of basis functions $|j, NR, JM\rangle$ given by

$$|j, NR, JM\rangle = \sum_{mM_R} \langle jm R M_R | JM\rangle |jm\rangle |NR M_R\rangle$$

where J and M represent the resultant angular momentum and its projection on the z axis. The off diagonal matrix elements of the total Hamiltonian in the basis vectors given above are only due to the interaction part given by

$$\begin{aligned} & \langle j', N'R', JM | H_{int} | j, NR, JM \rangle \\ &= -\frac{1}{2} \zeta \hbar \omega (-1)^{J+j+j'-1/2} \\ & \times \left\{ (2j+1)(2j'+1) \right\}^{1/2} \left\{ \begin{matrix} j & R & J \\ R' & j' & 2 \end{matrix} \right\} \left(\begin{matrix} j & 2 & j \\ -1/2 & 0 & 1/2 \end{matrix} \right) \\ & \times \{ (-1)^{R'} \langle N'R' || b || NR \rangle \\ & + (-1)^R \langle NR || b || N'R' \rangle \} \delta_{l+l'}^{\text{even}} \end{aligned}$$

The reduced matrix elements of the annihilation operator are defined according to Racah's definition and are larger than those used by Choudhury by a factor $\sqrt{2R'+1}$. The interaction matrix elements vanish for $\Delta N > 1$ and $\Delta R > 2$. The diagonal matrix elements are given by

$$\begin{aligned} & \langle j', N'R', JM | H_c + H_p | j, NR, JM \rangle \\ &= \hbar \omega \left\{ N + 5/2 + \frac{E_j}{\hbar \omega} \right\} \delta_{N'N} \delta_{R'R} \delta_{j'j} \end{aligned}$$

Thus matrix for any given angular momentum space can be set up and diagonalised to yield eigen values and eigen vectors. The absolute values of energies of the different angular momentum states can be worked out assuming the lowest energy of one of angular momentum states to correspond to zero. In this matrix diagonalisation procedure $\zeta \hbar \omega$ and E_j may be treated as parameters. Several investigators assumed $\hbar \omega$ to represent the one phonon energy of the even-even core nucleus, while in some studies an average of $\hbar \omega$ in the region was employed. The justification for the latter procedure was based on the assumption that the properties of the preceding

even-even nucleus may not fully represent the core properties. The single particle energies were likewise selected in some studies from the experimental spectra in the neighbouring region. The usual range of ζ in several cases was from 1 to 4. For each value of ζ in this region and for each of the other parameters as outlined above matrix diagonalisation is carried out and the resultant eigen values are compared with the experimental spectra. From the many sets of the parameters over which the procedure is repeated the best set is selected by a χ^2 minimisation requirement. The eigen vectors corresponding to the best set of parameters can be employed to estimate the static and dynamic electromagnetic moments, most useful among them being $B(E2)$, $B(M1)$ values and the magnetic dipole moment and the electric quadrupole moment. The relevant operators for the estimation of these quantities using the eigen vectors and the explicit expressions for the estimation of these quantities are detailed in the paper of Hyde and Brussard⁽¹⁶⁾.

The regions extensively studied in this model are those in the beginning of the 5th major shell of protons and the end of the shell for the neutrons. The nuclei of this description are in the mass region 100 to 150 and are typical spherical nuclei. One of the essential prerequisites for the application of this model is that the neighbouring even-even nuclei should exhibit typical vibrational spectra and for nuclei cited above this criterion is very well fulfilled. There are also other regions in which attempts were made to apply this model, nuclei with mass number greater than 190 being typical example. In some of the investigations it was noticed that a better fit with the experimental results could be obtained by considering the anharmonicities in the phonon spectra of the core nucleus. Instead of evaluating the diagonal elements for different phonon numbers based on the same value of $\hbar \omega$, the actual values of the energies of the phonon levels could be employed. This procedure, however, is restricted to phonon number less than 2 inasmuch as extensive studies were carried out to locate the vibrational levels upto two phonons only. The angular momentum states representing higher number of phonons are less well-established. In addition considerable admixtures of other types of excitation are expected in these cases, so that

interpretation by invoking pure vibrational excitation may not be meaningful.

In a recent work⁽²⁵⁾ the scope of the intermediate coupling model calculations is extended by including multiple particle configuration. Thus the energy levels of odd-mass cesium nuclei were interpreted based on three hole configurations in $g_{7/2}$ orbit coupled to phonon excitations. Out of the many possible angular momentum states of such a configuration only a few angular momentum states are considered to limit the size of the matrix. In some of the calculations for similar reasons phonon number upto 2 alone were considered. Such a procedure may be justified if only low energy excited states are considered and the coupling strength is small enough to justify the neglecting of a thorough mixing of phonons. Although a considerable measure of success was achieved in the work of several investigators, it should be remembered that the approach is phenomenological and is not expected to yield exact solutions. Nevertheless a systematic study of nuclei in different regions to determine the dependence of the coupling strength on nucleon numbers is expected to yield useful information.

TABLE (4.1)

Reduced E2 Matrix Elements for Tl^{203} and
neighbouring even-even Nuclei

Nucleus	E2 Transitions		$T_i \rightarrow f / E^2$ in $\text{sec}^{-1} \text{MeV}^{-2}$
$^{203}_{81}Tl$	$3/2 \rightarrow 1/2$	278 keV	0.8×10^{12}
	$5/2 \rightarrow 1/2$	680 keV	0.8×10^{12}
$^{208}_{81}Tl$	$3/2 \rightarrow 1/2$	205 keV	0.6×10^{12}
	$5/2 \rightarrow 1/2$	615 keV	0.5×10^{12}
$^{208}_{82}Pb$	$2 \rightarrow 0$	803 keV	0.3×10^{12}
$^{204}_{80}Hg$	$2 \rightarrow 0$	430 keV	0.6×10^{12}
$^{202}_{80}Hg$	$2 \rightarrow 0$	440 keV	1.3×10^{12}

TABLE (4.2)

Reduced E2 Matrix Elements for Ag¹⁰⁷, Ag¹⁰⁹
and neighbouring even-even Nuclei

Nucleus	E2 Transitions	$T_{i \rightarrow f}/E^5$ in sec ⁻¹ MeV ⁻³
$^{107}_{47}\text{Ag}_{60}$	$3/2^- \longrightarrow 1/2^-$ 324 keV	1.4×10^{12}
	$5/2^- \longrightarrow 1/2^-$ 423 keV	1.4×10^{12}
$^{109}_{47}\text{Ag}_{62}$	$3/2^- \longrightarrow 1/2^-$ 309 keV	1.8×10^{12}
	$5/2^- \longrightarrow 1/2^-$ 416	1.6×10^{12}
$^{106}_{46}\text{Pd}_{60}$	$2^+ \longrightarrow 0^+$ 513 keV	1.3×10^{12}
$^{108}_{48}\text{Cd}_{60}$	$2^+ \longrightarrow 0^+$ 630 keV	1.6×10^{12}
$^{108}_{46}\text{Pd}_{62}$	$2^+ \longrightarrow 0^+$ 433 keV	1.0×10^{12}
$^{110}_{48}\text{Cd}_{62}$	$2^+ \longrightarrow 0^+$ 656 keV	1.2×10^{12}

TABLE (4.3)

A ²	0.96	0.933	0.91	Exp.
Q [g.s] barn	$+0.47 \pm 0.18$	$+0.59 \pm 0.18$	$+0.67 \pm 0.18$	± 0.56
δ^2 (77 keV)	0.18 ± 0.06	0.24 ± 0.06	0.03 ± 0.06	0.11
$-\mu$ [g.s](nm)	0.139 ± 0.002	0.138 ± 0.002	0.137 ± 0.002	0.144
$\mu(\frac{1}{2}^+)$ (nm)	0.59 ± 0.02	0.45 ± 0.04	0.40 ± 0.04	0.37 ± 0.05
δ^2 (191 keV)	0.04 ± 0.02	0.05 ± 0.03	0.06 ± 0.03	0.33 ± 0.26
E2(3/2) ₂ →(3/2) ₁	0.006 ± 0.04	0.025 ± 0.04	0.10 ± 0.04	0.08 ± 0.01

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